Kinetic-MHD hybrid simulations of energetic-particle driven instabilities

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12th ITER International School

“The Impact and Consequences of Energetic Particles on Fusion Plasmas”
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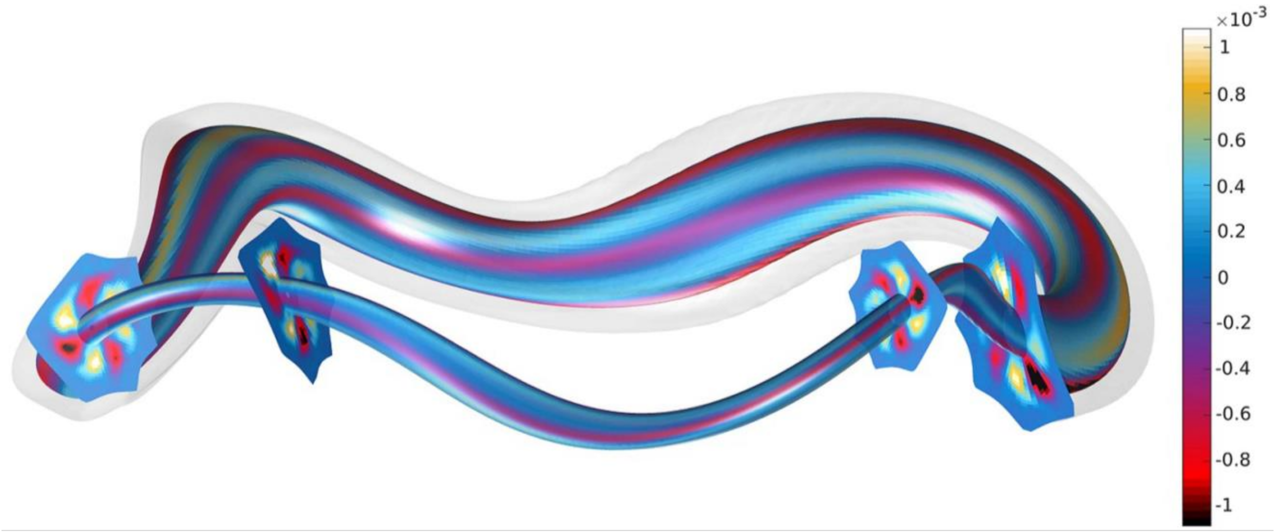


[P. Adulsiriswad+, NF 60, 096005 (2020)]

Outline

- Energetic particles (EPs） and Alfvén eigenmodes (AEs) in fusion plasmas
- Resonance condition, conserved quantity, and inverse Landau damping
- Phase space islands created by particle trapping and higher-order islands
- Nonlinear evolution of a bump-on-tail instability and frequency chirping
- Kinetic-MHD hybrid simulation
- Hybrid simulation for EP and MHD
  - Nonlinear MHD effects and zonal flow generation
  - Validation on DIII-D experiments (fast ion profile flattening and stiffness, electron temperature fluctuations)
  - AE burst and critical fast-ion distribution (profile resiliency)

Energetic particle confinement is important for fusion energy

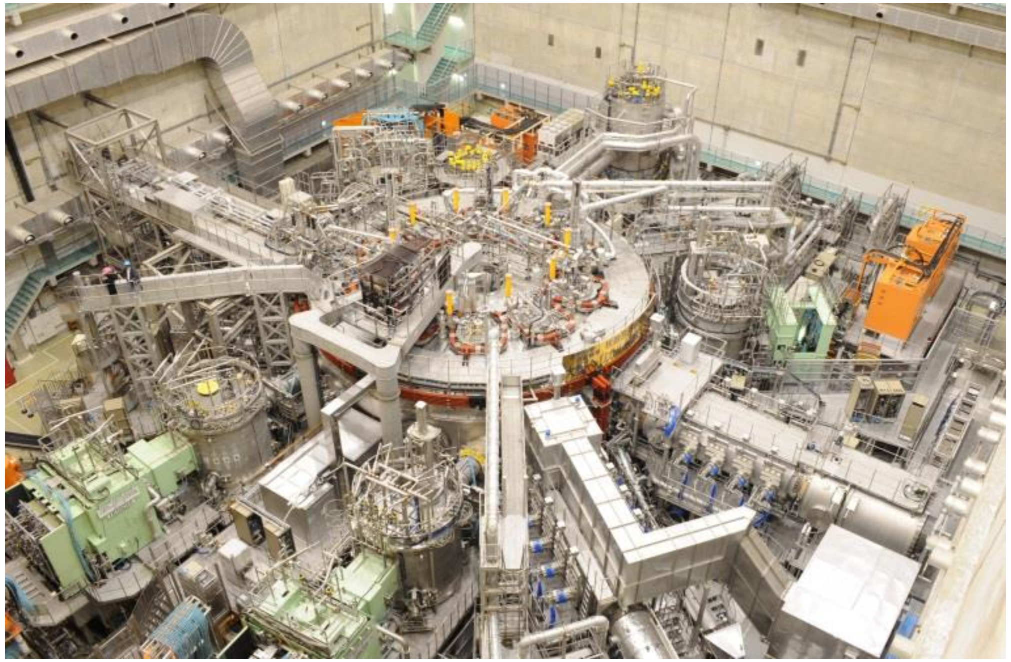
- Nuclear fusion: safe and environmentally friendly energy source in the next generation
- Fusion reaction of deuterium (D) and tritium (T) in high temperature plasmas

$$D + T \rightarrow {}^{4}He \text{ (helium, 3.5 MeV)} + n \text{ (neutron, 14 MeV)}$$

- Energetic He (alpha) heats the plasma
- Confinement of energetic alpha particles are important for the sustainment of high temperature (>10keV)

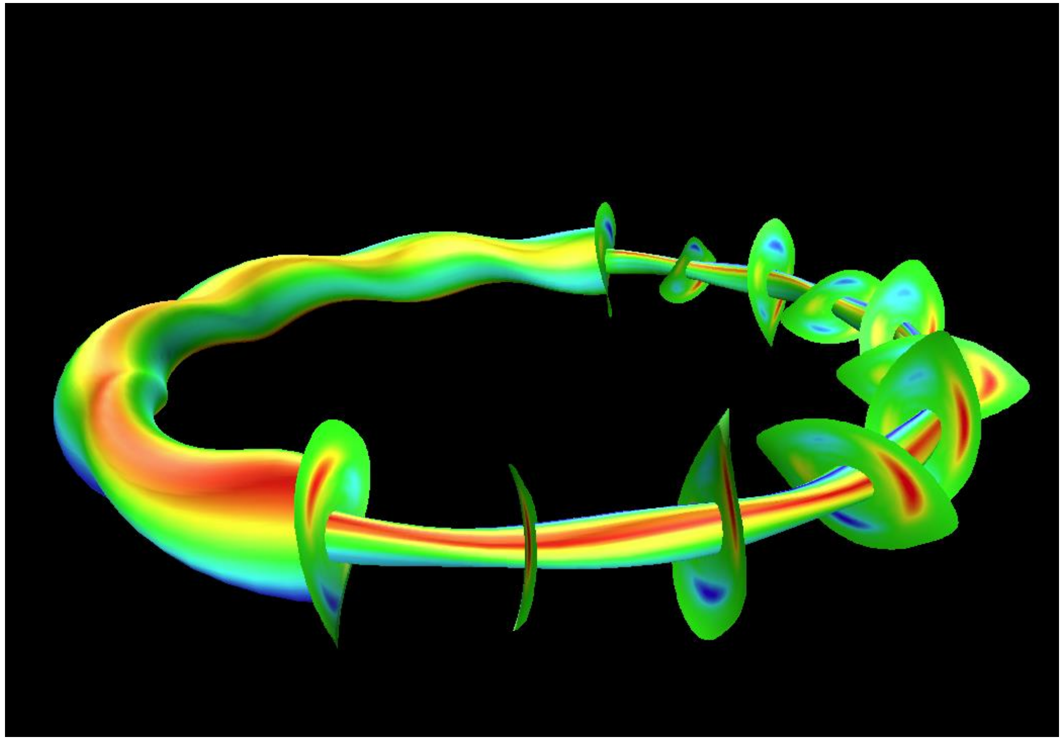
Energetic particles in fusion plasmas

- Alpha particle born from D-T reaction
  D+T -> He^4 (3.5MeV) + n (14MeV)
- Neutral beam injection (NBI)
- Ion cyclotron heating (ICH)
- Electron cyclotron heating (ECH)

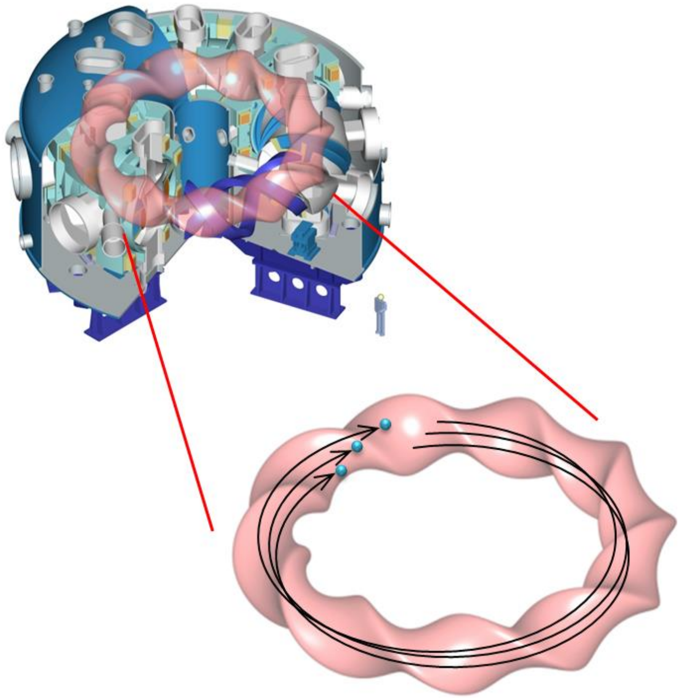


Large Helical Device (LHD)

Interaction between Alfvén eigenmodes (AEs) and energetic particles

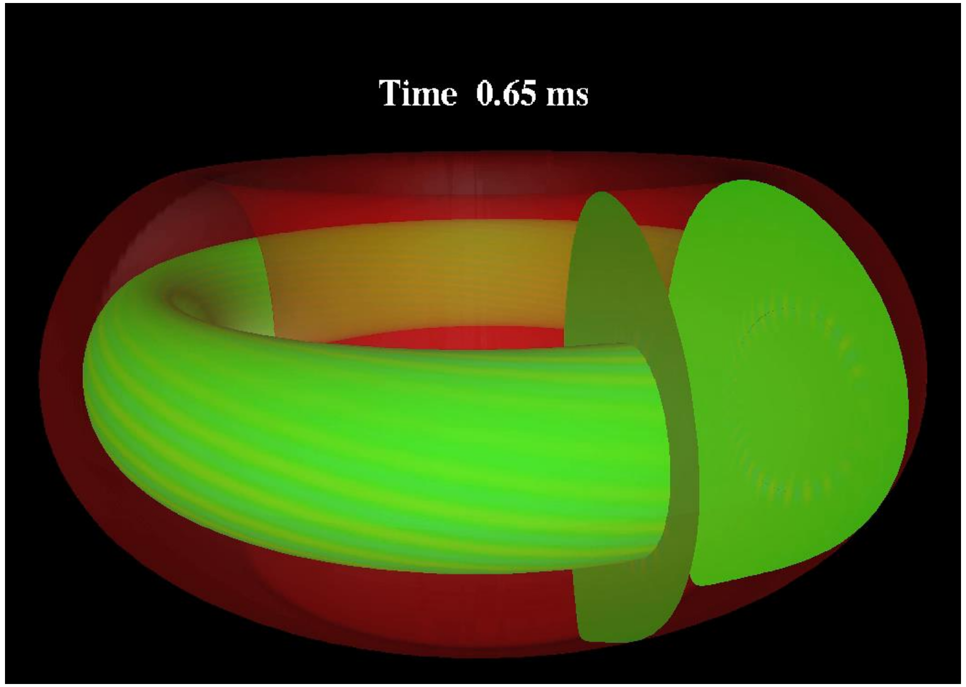


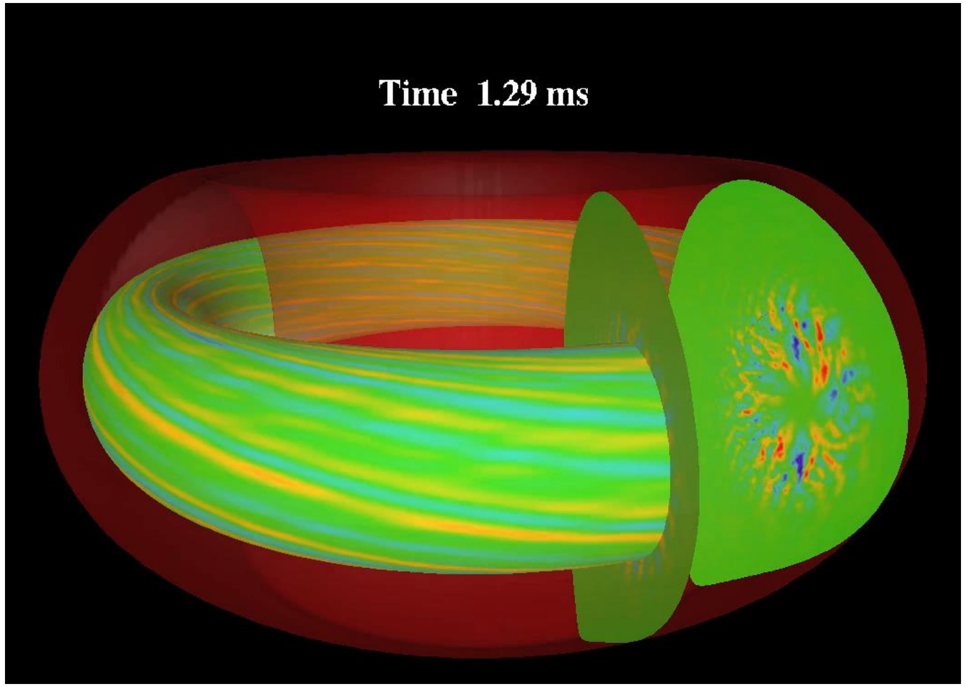
Alfvén eigenmode (magnetohydrodynamic oscillations) in LHD.



Energetic particles circulating inside the plasma interact with and destabilize AEs.

Time evolution of Alfvén eigenmodes in an ITER steady state scenario





Y. Todo and A. Bierwage,
Plasma and Fusion Research **9**, 3403068 (2014)

Is the interaction between EP and AE an important research subject in (plasma) physics?

- This is a problem of inverse Landau damping
  - with MHD waves
  - in 3D magnetically confined plasma (complicated particle orbit)
  - with non-uniform spatial distribution (spatial distribution is important as well as velocity distribution)
  - extended with source and sink (open system, EP distribution formation process, steady and intermittent evolution)

INTERACTION BETWEEN EP AND AE

Resonance condition in toroidal plasmas (1)

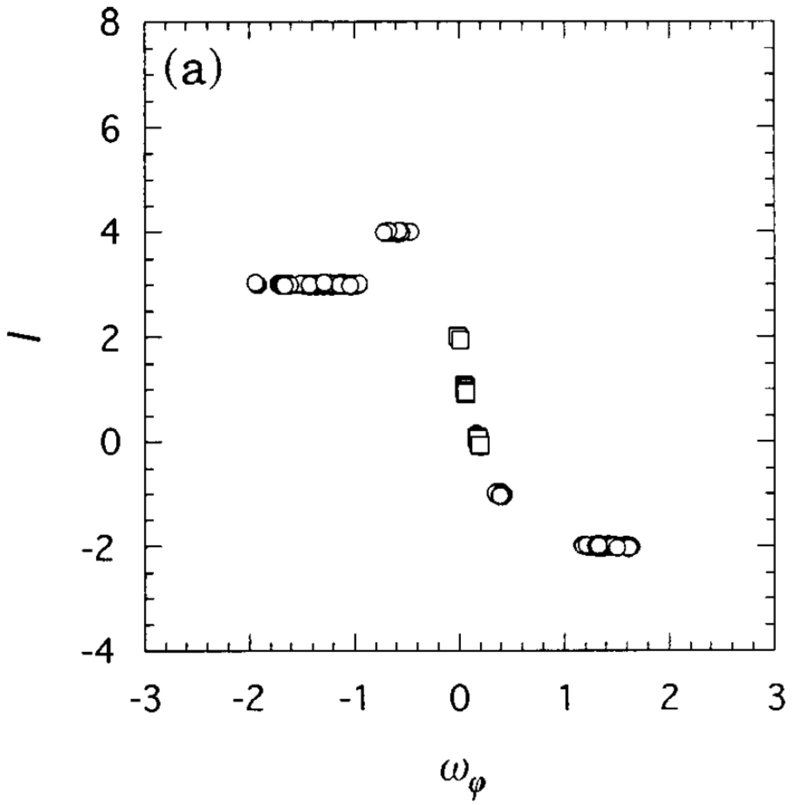
- When a resonant particle passes one round in the poloidal angle, the phase of the AE should change by a multiple of 2π.
  This gives the resonance condition:
  $\omega * T_\theta - n * \Delta\varphi = L * 2\pi$
  T_θ: time for the particle to pass one round in the poloidal angle
  $\Delta\varphi$: toroidal angle which the particle passes in T_θ
  L: integer

- $\omega - n * \omega_\varphi - L * \omega_\theta = 0$
  $\omega_\varphi = \Delta\varphi / T_\theta$
  $\omega_\theta = 2\pi / T_\theta$
  or $L = (\omega - n * \omega_\varphi) / \omega_\theta$

Resonance = When a particle passes one round in the poloidal angle, the phase of the wave is the same as that at the previous visit.

Y. Todo, Reviews of Modern Plasma Physics **3**, 1 (2019)

Resonance condition in toroidal plasmas (2)



Particles strongly interacting with an Alfven eigemode in the simulation.

Integer => Resonance

Large delta-f particles interacting with a TAE
Vertical axis: $(\omega - n * \omega_\varphi) / \omega_\theta$
Horizontal axis: ω_φ
[Todo and Sato, Phys. Plasmas **5**, 1321 (1998)]

Higher-order resonance

- When a resonant particle passes K rounds in the poloidal angle, the phase of the AE should change by a multiple of 2π.
  This gives the higher-order resonance condition:
  $K * (\omega * T_\theta - n * \Delta\varphi) = L * 2\pi$
  T_θ: time for the particle to pass one round in the poloidal angle
  $\Delta\varphi$: toroidal angle which the particle passes in T_θ
  L: integer

- $\omega - n * \omega_\varphi - (L/K) * \omega_\theta = 0$
  $\omega_\varphi = \Delta\varphi / T_\theta$
  $\omega_\theta = 2\pi / T_\theta$
  or $L/K = (\omega - n * \omega_\varphi) / \omega_\theta$

- Higher-order resonance (fractional resonance) has no effect on linear stability, but may have substantial effects for finite amplitude wave.

Constants of motion in toroidal plasmas (1)

- In axisymmetric (independent of toroidal angle φ) equilibrium (time-independent) fields:

  - energy E
  - magnetic moment μ
  - toroidal canonical momentum $P_\varphi = e_h\Psi + m_h R v_\varphi$ are constant along particle orbit
    (Ψ is poloidal magnetic flux, e_h and m_h are charge and mass)

Constants of motion in toroidal plasmas (2)

- In the presence of a wave with angular frequency ω and toroidal mode number n:

  - μ is conserved if $\omega \ll \Omega_h = e_h B/m_h$
  - neither energy nor toroidal canonical momentum is conserved.
  - however, their combination $E' = E - \omega P_\varphi/n$ is conserved.

E' is conserved during the wave-particle interaction in axisymmetric equilibrium (1)

Energy and toroidal momentum evoution with

equilibrium field Hamiltonian H_0 and wave Hamiltonian H_1

$$\frac{dE}{dt}=\frac{\partial H}{\partial t}=\frac{\partial}{\partial t}\left(H_0+H_1\right)=\frac{\partial}{\partial t}H_1$$

$$\frac{dP_\varphi}{dt}=-\frac{\partial H}{\partial \varphi}=-\frac{\partial}{\partial \varphi}\left(H_0+H_1\right)=-\frac{\partial}{\partial \varphi}H_1$$

because $\frac{\partial}{\partial t}H_0=0$ (equilibrium)

and $\frac{\partial}{\partial \varphi}H_0=0$ (axisymmetric).

E’ is conserved during the wave-particle interaction in axisymmetric equilibrium (2)

Suppose the wave amplitude is constant,

H_1 is written in cylindrical coordinates (R, φ, z)

$$H_1 = \hat{H}_1(R,z)e^{in\varphi - i\omega t}$$

$$\frac{dE}{dt} = \frac{\partial H_1}{\partial t} = -i\omega \hat{H}_1(R,z)e^{in\varphi - i\omega t}$$

$$\frac{dP_\varphi}{dt} = -\frac{\partial H_1}{\partial \varphi} = -in\hat{H}_1(R,z)e^{in\varphi - i\omega t}$$

Then, $\dfrac{dE'}{dt} = \dfrac{d}{dt}\left(E - \dfrac{\omega}{n}P_\varphi\right) = 0$ is satisfied.

Conservation of E’ suggests …

In wave-particle interaction in tokamak plasmas,

the conservation of E' leads to

$$\frac{\delta E}{\omega} = \frac{\delta P_\varphi}{n} \approx \frac{e_h \delta\psi}{n}$$

Energy transfer between wave and particle (δE),

and change in poloidal mangetic flux ($\delta\psi$)

(= radial location; spatial transport) are related to each other.

This suggests qualitatively

for high ω (such as ICRF): δE is important — Energy transfer, wave heating

for low ω and high n (such as ITG): $\delta\psi$ is important — Transport in radial direction

Why are AEs destabilized by energetic particles? (1)

Q1: Can particles interact with ideal MHD modes with $E_{//} = 0$?

A1: in toroidal plasmas, grad-B and curvature drifts -> $\mathbf{v}_{\perp}$ -> energy transfer through $e_h \mathbf{v}_{\perp} \cdot E_{\perp}$

Q2: Slowing down distribution and Maxwell distribution have negative gradient in energy

$$\frac{\partial f}{\partial E} < 0$$

This leads to the stabilization of AEs due to Landau damping.

Why are AEs destabilized by energetic particles? (2)

We should consider the derivative

keeping $E' = E - \frac{\omega}{n} P_\varphi$ =constant,

$$\left.\frac{\partial f}{\partial E}\right|_{E'} = \frac{\partial f}{\partial E} + \frac{n}{\omega}\frac{\partial f}{\partial P_\varphi}$$

The toroidal momentum is $P_\varphi = e_h \psi + m_h R v_\varphi$.

If we apporixmate $P_\varphi \cong e_h \psi$,

$$\frac{n}{\omega}\frac{\partial f}{\partial P_\varphi} \cong \frac{n}{\omega}\frac{\partial f}{e_h \partial \psi} = \frac{n}{\omega}\frac{1}{e_h R B_\theta}\frac{\partial f}{\partial r}$$

With $B_\theta = \frac{\tau r B}{qR}$ $(B > 0,\ \tau = -1 \text{ or } 1)$

$$\frac{n}{\omega}\frac{\partial f}{\partial P_\varphi} \cong \frac{n}{\omega}\frac{\tau q}{e_h B}\frac{\partial f}{r \partial r}$$

Why are AEs destabilized by energetic particles? (3)

Introducing "tempereature" T which replaces energy derivative

$$\frac{\partial f}{\partial E} = -\frac{f}{T}, \text{ and } w_* = \frac{t\, qT}{e_h B}\frac{\partial \ln f}{r\,\partial r},$$

$$\left.\frac{\partial f}{\partial E}\right|_{E'} = -\frac{f}{T}\left(1 - \frac{n}{w} w_*\right)$$

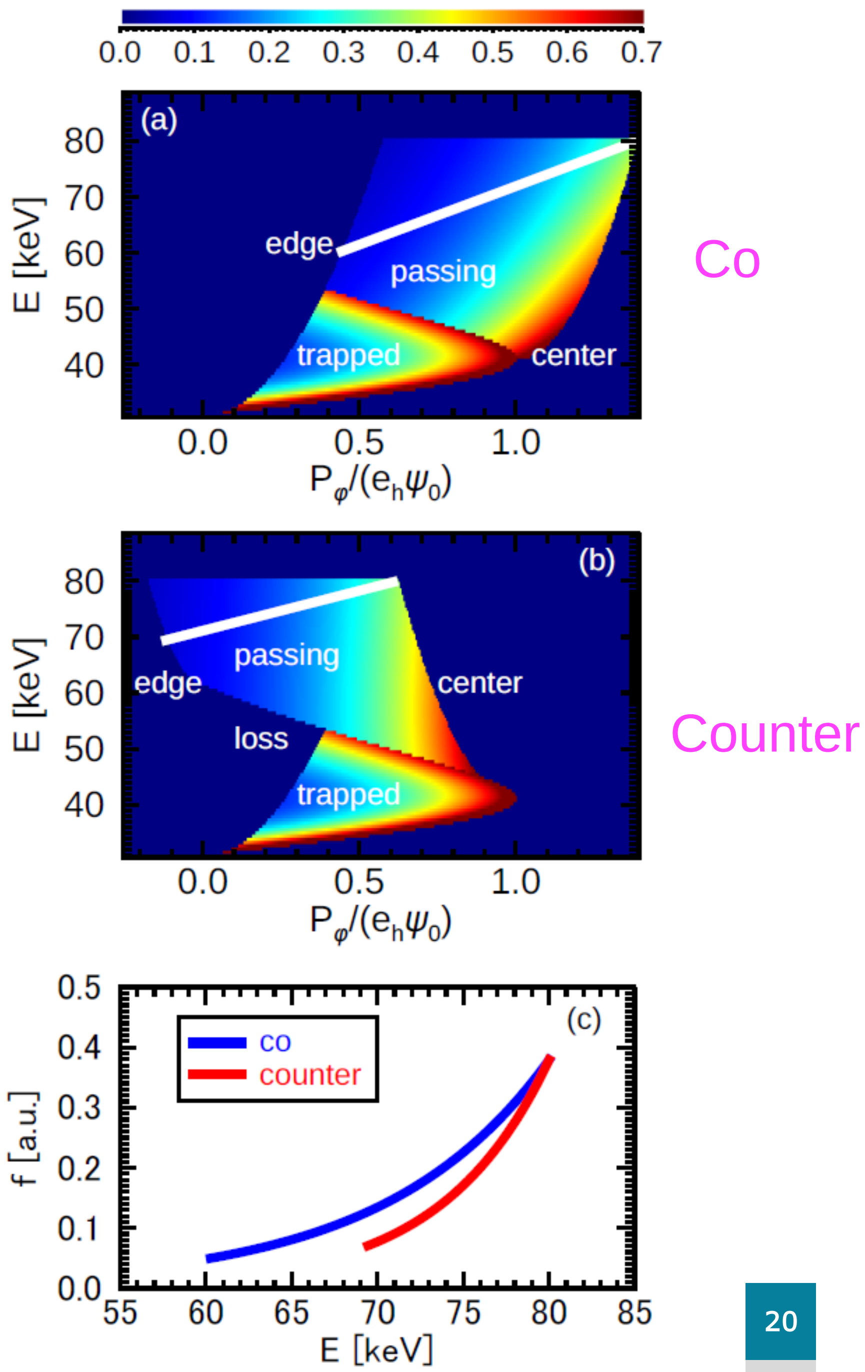
When the radial gradient of f is sufficiently large,

the second term $\frac{n}{w} w_*$ makes $\left.\frac{\partial f}{\partial E}\right|_{E'} > 0$ to destabilize the AE.

This also determines the sign of n / w, i.e. the toroidal propagation direction of the AE depending on the sign of $\frac{\partial f}{\partial r}$.

Why are AEs destabilized by energetic particles? (4)

- Fast-ion distribution function in a tokamak plasma: f（P_φ,E）, μ=const.
- R_0=1.8m, a=0.6m
- B_0=2T, q_0=1.1, q_{edge}=3.0
- Deuterium, isotropic slowing-down distribution
- E_b=80keV, E_c=30keV
- Exp [$-P_\varphi/0.4\Psi_0$]
- White lines: E'=const
- AE with toroidal mode number n=4, freq.=70kHz



Y. Todo, Reviews of Modern Plasma Physics **3**, 1 (2019)

Why are AEs destabilized by energetic particles? (5)

For $n = 0$ modes, the energetic particle spatial gradient does not destabilize the AE modes.

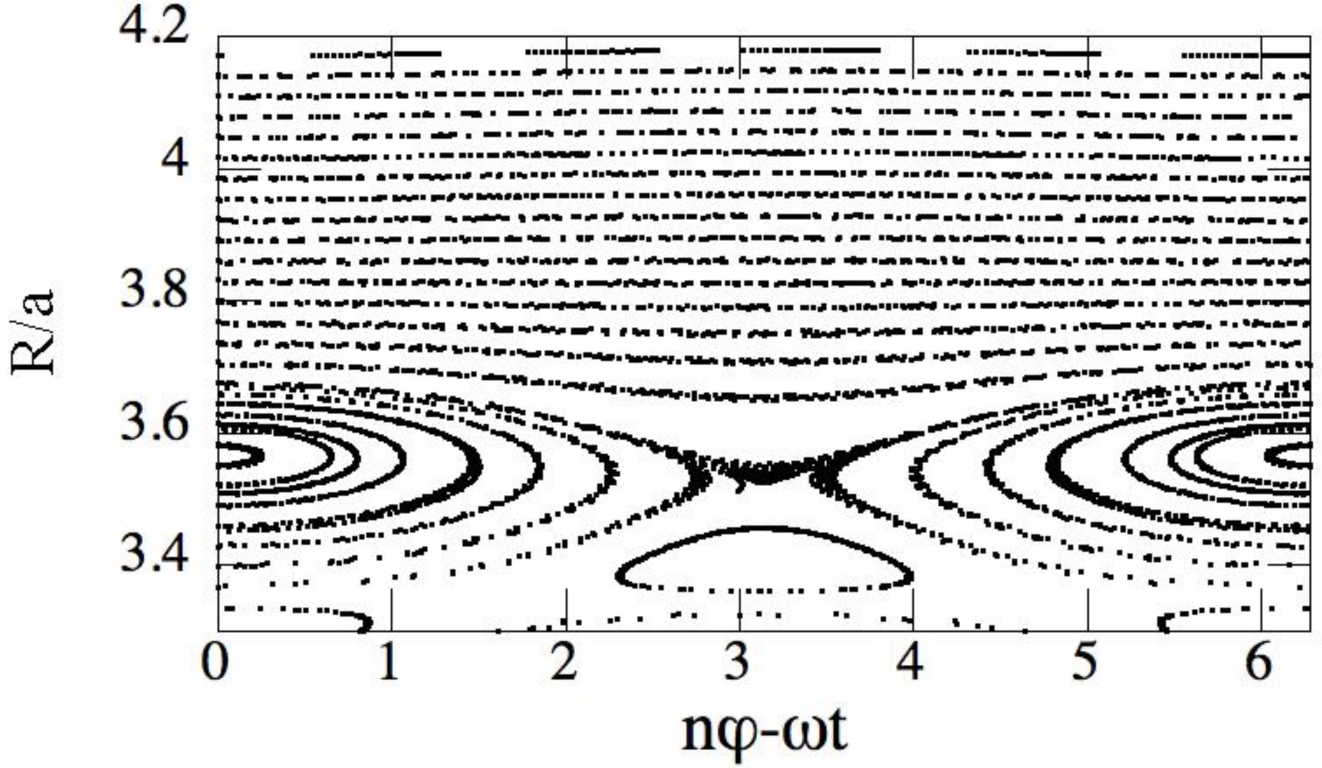
However, when f is not isotropic in velocity space and depends on pitch angle variable $\Lambda = \mu B/E$, $f = f(E,\Lambda)$

$$\frac{\partial f}{\partial E} = \left.\frac{\partial f}{\partial E}\right|_{\Lambda} + \frac{\partial \Lambda}{\partial E}\left.\frac{\partial f}{\partial \Lambda}\right|_{E}$$

The second term on the R.H.S. can lead to destabilization of $n = 0$ modes such as GAM.

EP PHASE SPACE STRUCTURE WITH AE

Poincaré plot of energetic-ion orbits in the presence of an AE with constant amplitude ($\delta B/B = 2\times10^{-3}$ at the peak)



Particles have the same m and $E' = E - \frac{\omega}{n} P_\varphi$.

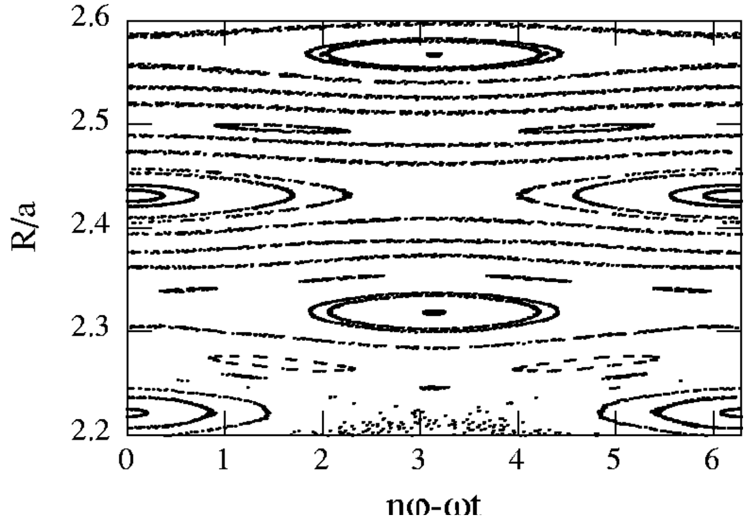
[Todo+, Phys. Plasmas 10, 2888 (2003)]

- An island structure is formed in the phase space.
- This is the region of particles trapped by the AE.

Higher-order islands and stochastic regions

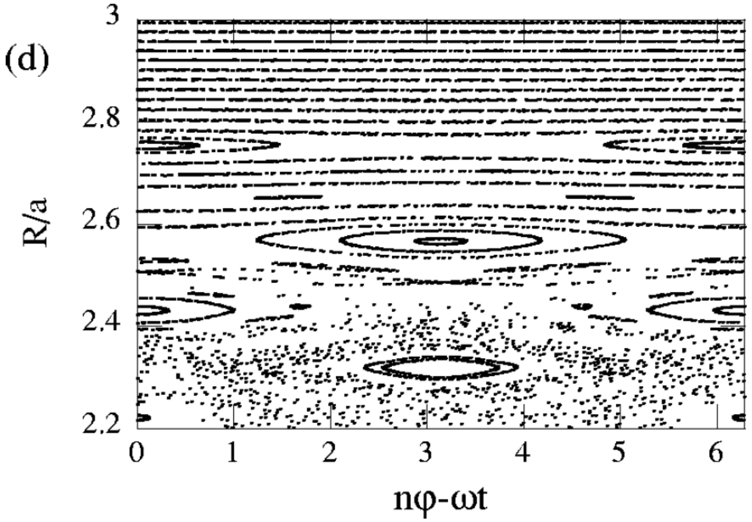
n=3 $\delta B/B=8\times10^{-4}$

many islands created by the higher-order resonances



n=3 $\delta B/B=2\times10^{-3}$

KAM surfaces disappear due to the overlap of the higher-order islands

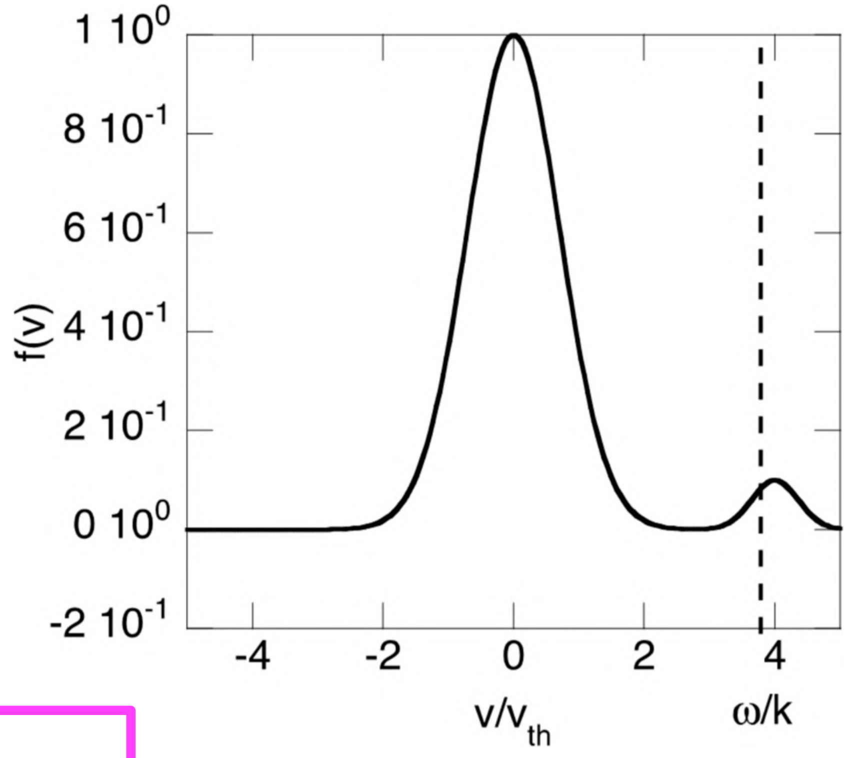


[Todo+, Phys. Plasmas **10**, 2888 (2003)]

NONLINEAR EVOLUTION OF BUMP-ON-TAIL INSTABILITY AND FREQUENCY CHIRPING

Bump-on-tail instability

- 1-dimensional electrostatic model
- The bump in the high energy region leads to an inverse Landau damping



We consider
- Intrinsic wave damping (γ_d)
- Collision (ν) = effective source and sink

- It was expected that once the instability is saturated by particle trapping, the AE will damp monotonically in time with the intrinsic damping rate.
- However, the story was different when the AE is close to the marginal stability …

Berk, Breizman, Pekker, PRL **76**, 1256 (1996)
Berk, Breizman, Petviashvili, Phys. Lett. A **234**, 213 (1997), **238**, 408(E) (1998)
Berk et al., Phys. Plasmas 6, 3102 (1999)

Various types of time evolution of wave amplitude (1-dimensional simulation of bump-on-tail instability)

γ_d=0, ν=0

particle trapping

$\gamma_L >> \gamma_d$, ν =0

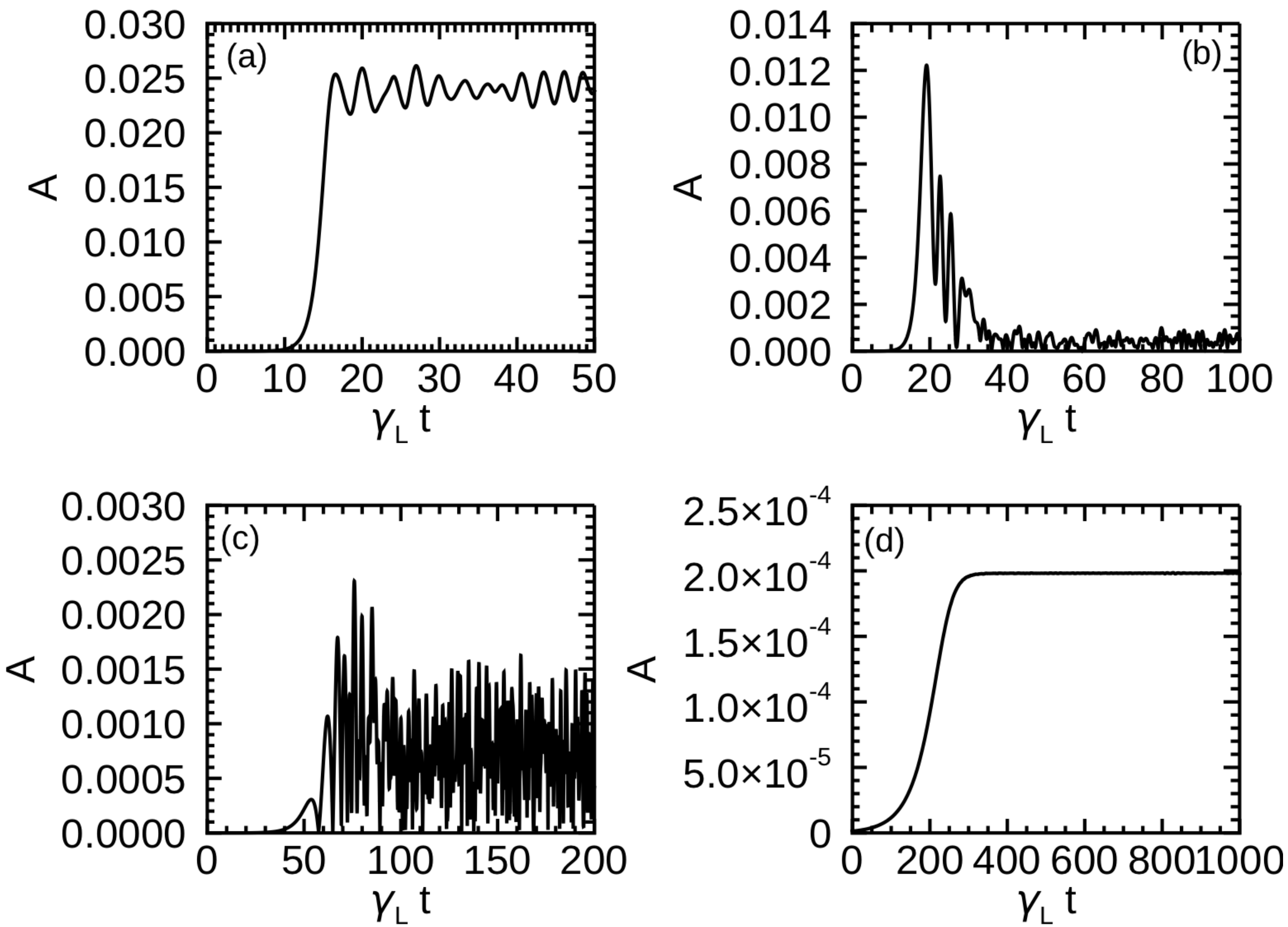
damping

$\gamma_L \sim \gamma_d$, ν=0

frequency chirping

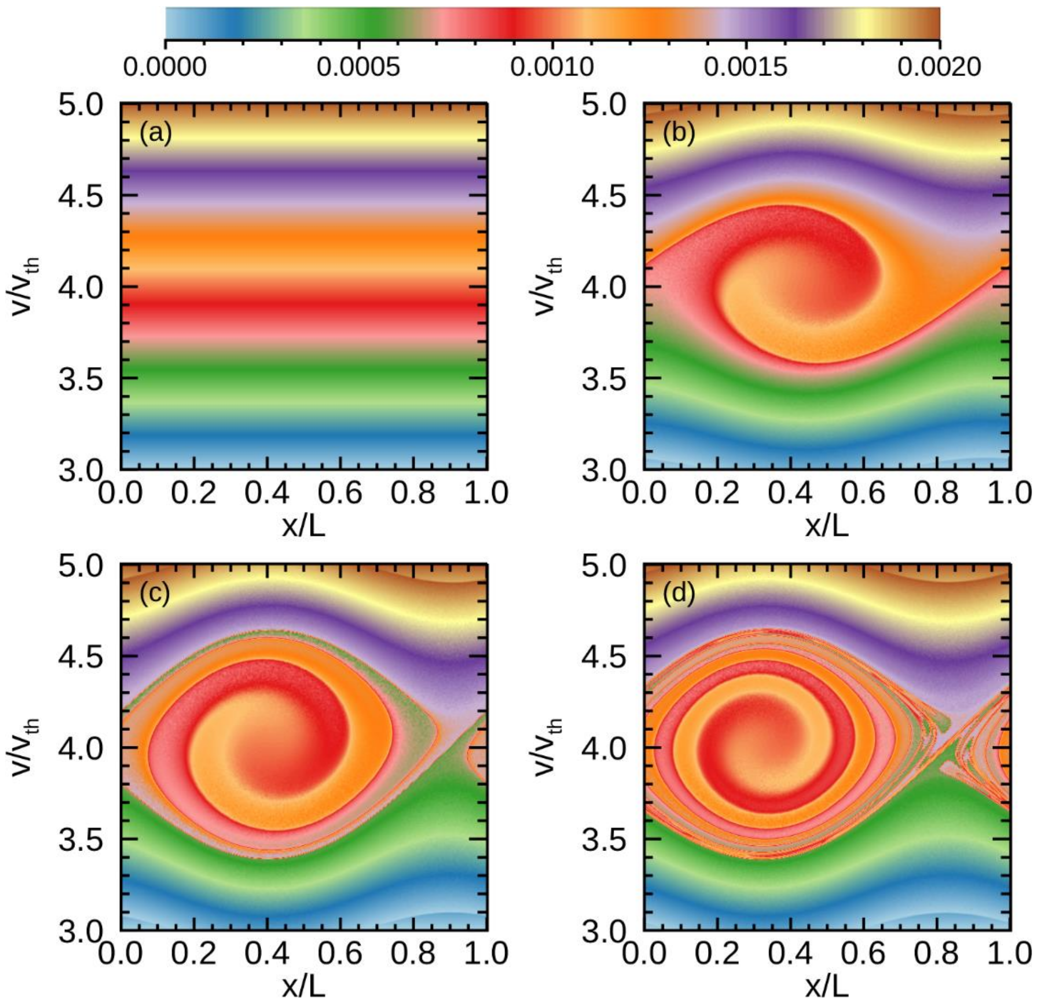
$\nu >> \gamma_L - \gamma_d$,

steady state



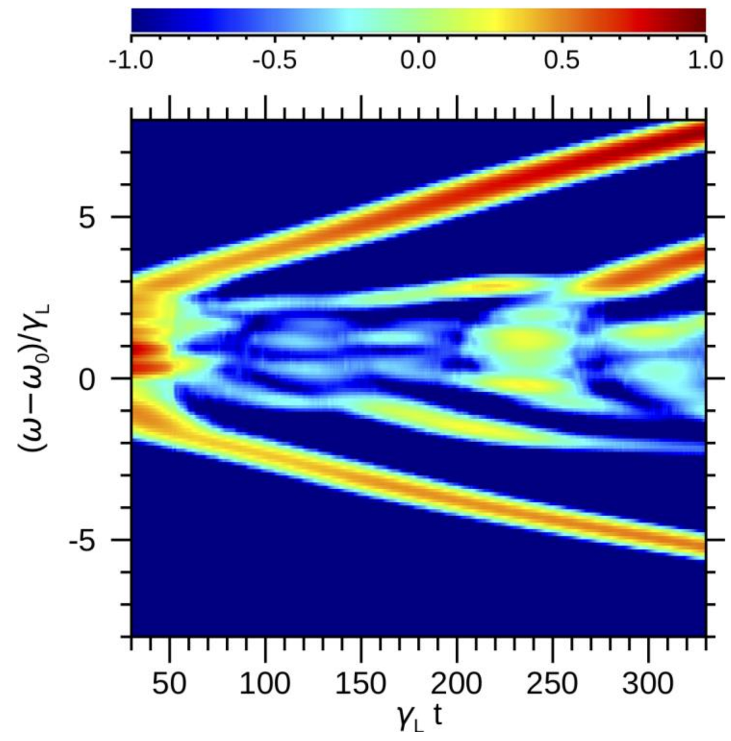
Y. Todo, Reviews of Modern Plasma Physics **3**, 1 (2019)

Distribution function evolution (γ_d=0, ν=0, particle trapping)

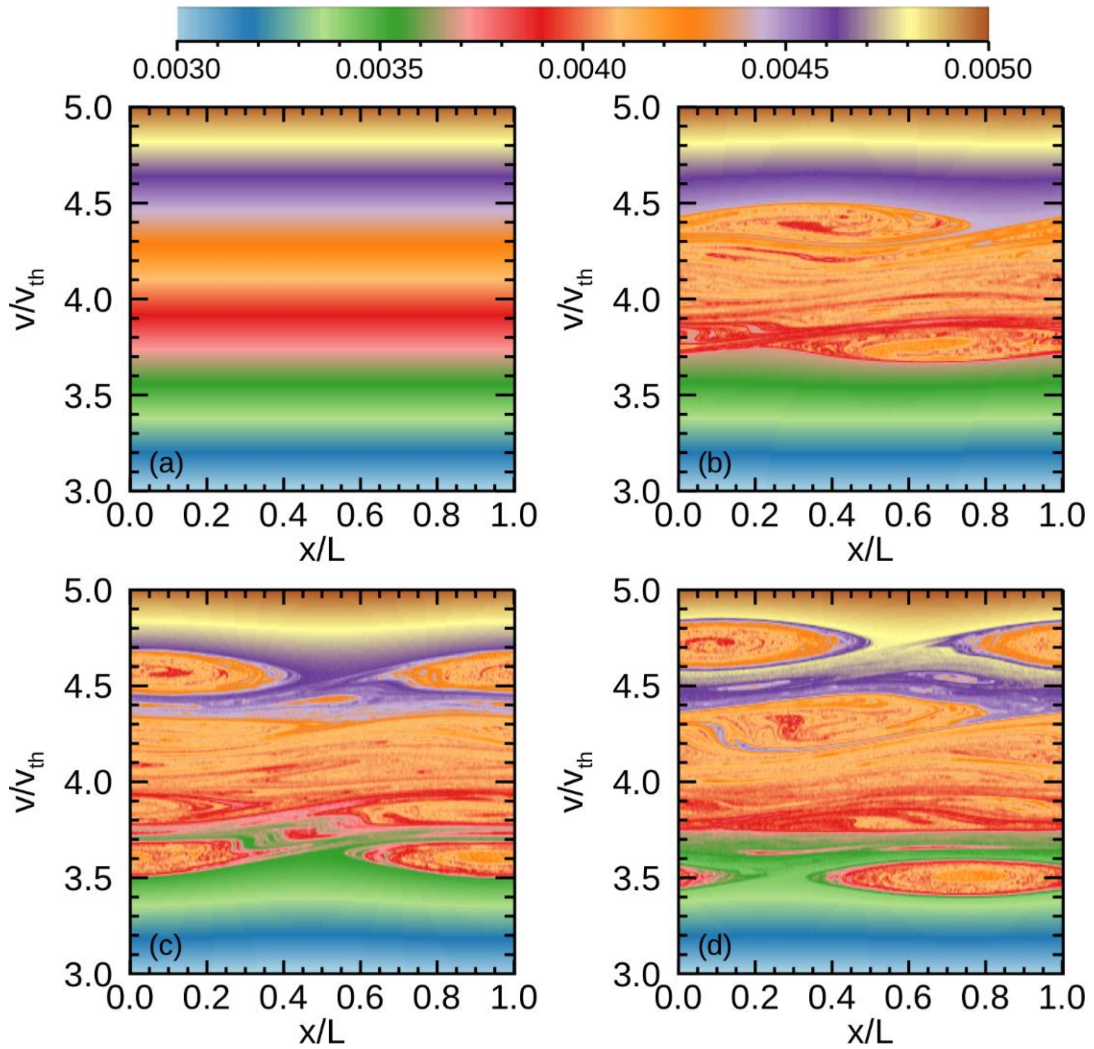


Y. Todo, Reviews of Modern Plasma Physics **3**, 1 (2019)

$\gamma_L \sim \gamma_d$, ν=0 frequency chirping



Frequency spectrum evolution



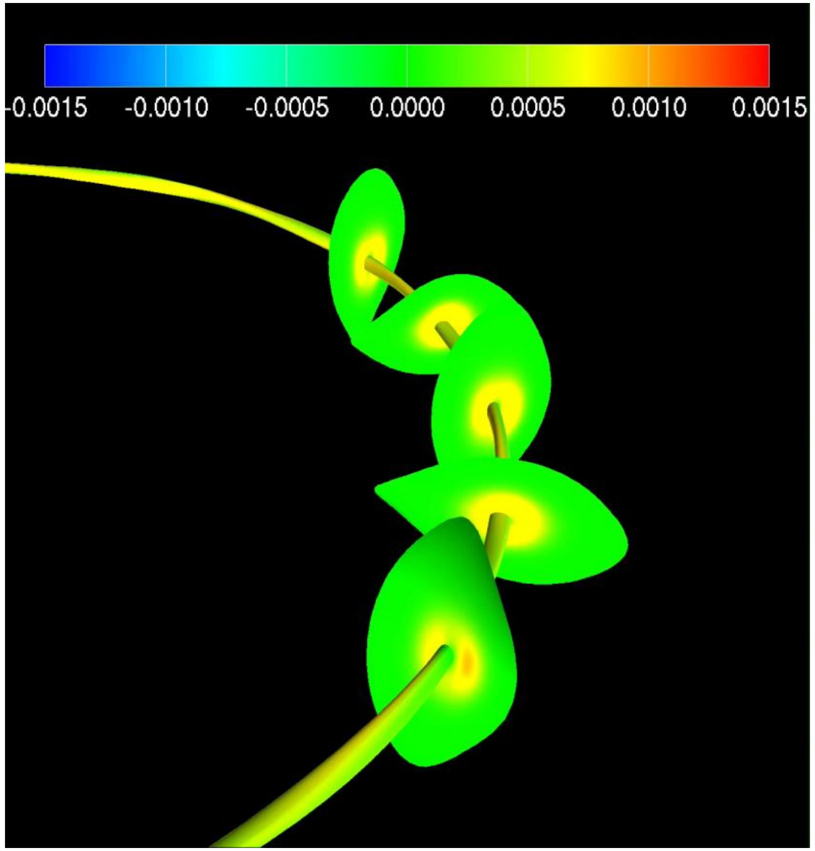
Distribution function evolution

Y. Todo, Reviews of Modern Plasma Physics **3**, 1 (2019)

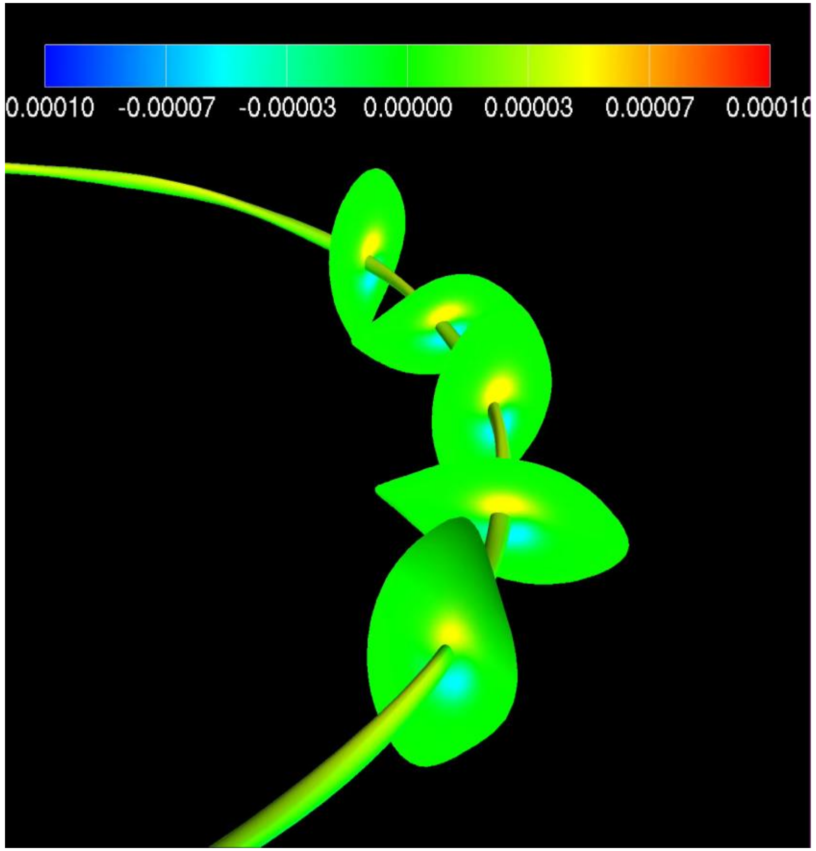
Explanation of frequency chirping

- Hole and clump are BGK modes.
- The structure of hole/clump and the frequency shift should be consistent with each other.
- The energy release due to the frequency chirping is balanced with the mode damping.

Simulation of energetic particle driven geodesic acoustic mode (EGAM) in LHD



MHD velocity



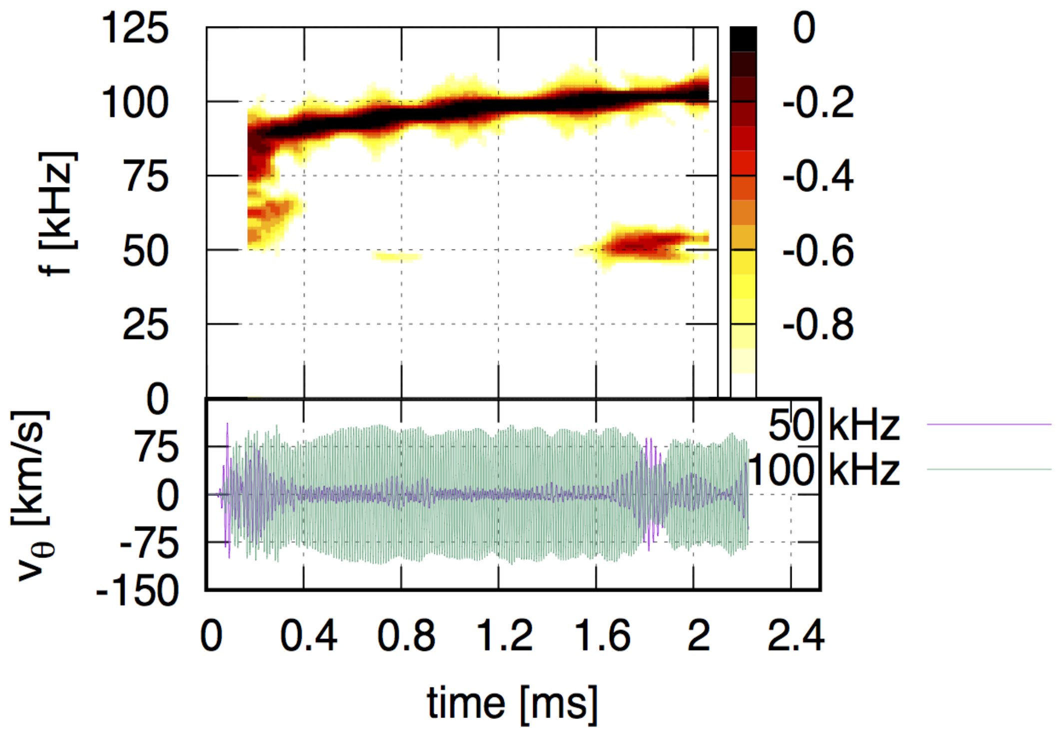
Plasma pressure

PLASMA SIMULATOR

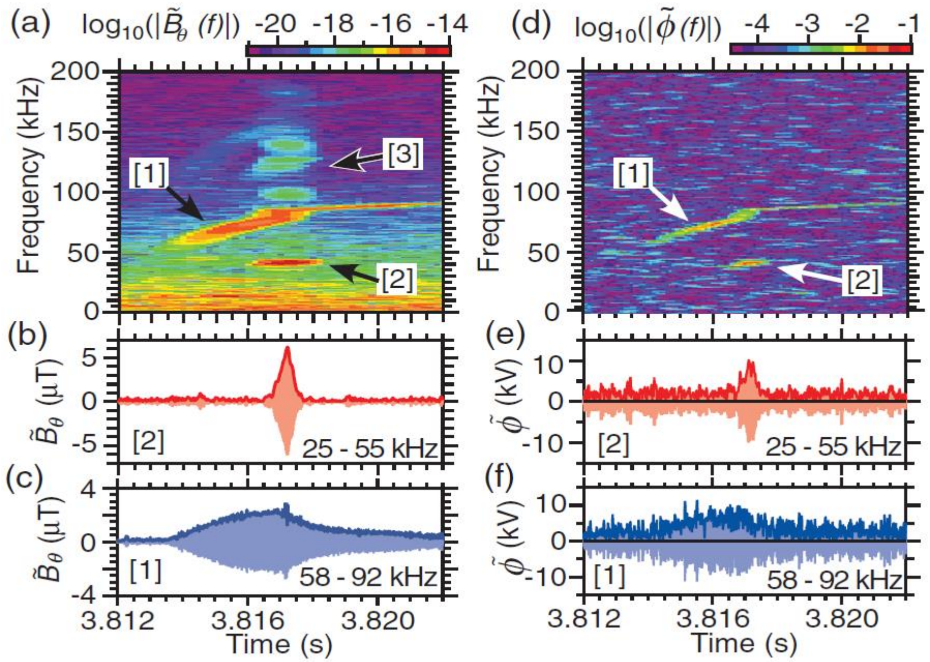
IFERC

K computer

Frequency chirping and sudden excitation of a half-frequency mode are reproduced by kinetic-MHD simulation



Simulation



LHD experiment
[T. Ido et al., PRL (2016),
M. Lesur et al., PRL (2016)]

[H. Wang et al., PRL **120**, 175001 (2018)]

KINETIC-MHD HYBRID SIMULATION

Initial value codes for the interaction between AEs and energetic particles

Method	AE modes	EP	Advantages	Codes
Kinetic-MHD hybrid simulation	MHD eq.	computational particles (or Vlasov eq. or gyrofluid eq.)	nonlinear MHD effects	M3D-C1 FAR3D HMGC HYMAGYC MEGA XTOR-K HYM
Reduced simulation	AEs given by linear analysis	computational particles	computationally less demanding	ORBIT HAGIS CKA-EUTERPE MEGA-R
Gyrokinetic simulation	computational particles for bulk plasma + GK Poisson eq. and Ampere eq.	computational particles (or Vlasov eq.)	fully kinetic effects	GTC ORB5 EUTERPE GEM GYRO GYSELA

Can MHD model AEs correctly?

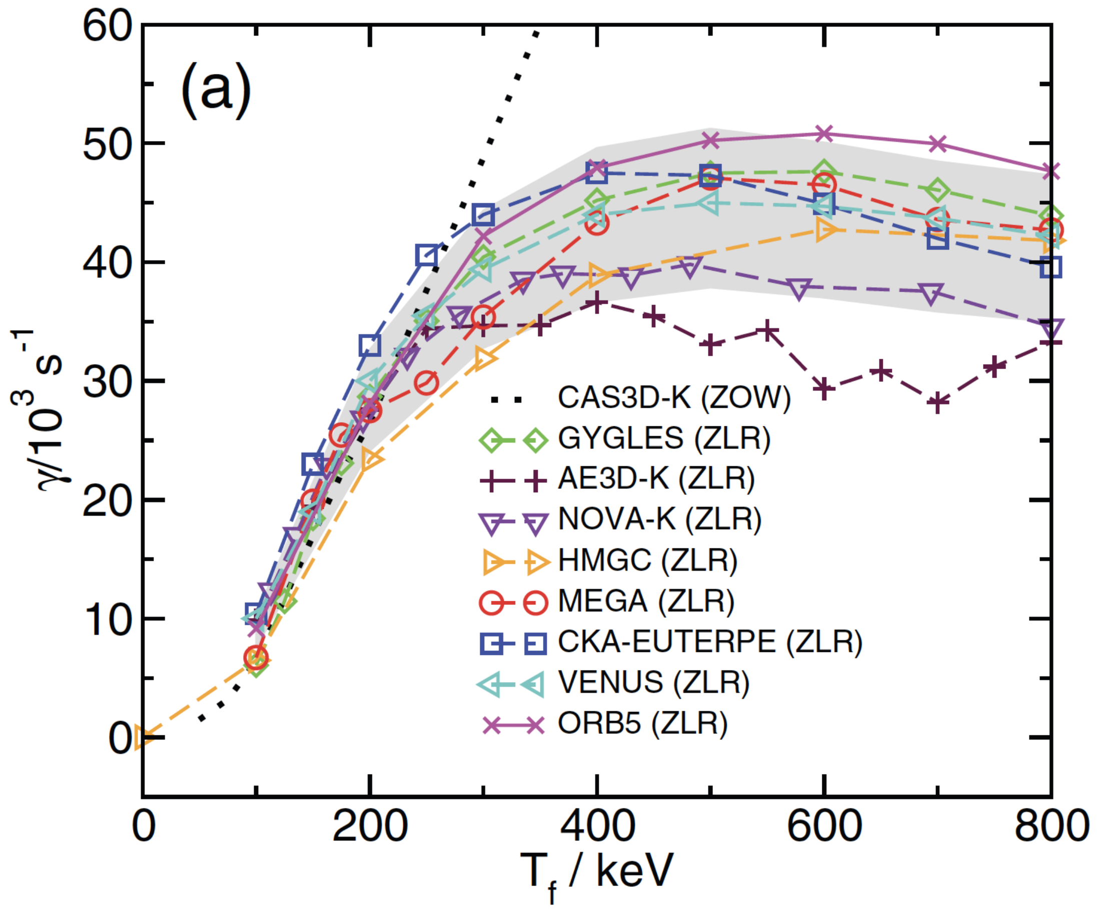
- Comparison in electron temperature fluctuation profile between MHD analysis (NOVA) and experiment on DIII-D [Van Zeeland (NF 2009)].
- “It is found that ideal MHD modelling of eigenmode spectral evolution, coupling and structure are in excellent agreement with experimental measurements.”

- => MHD is fine to model AEs !

M.A. Van Zeeland et al.,
Nuclear Fusion **49**, 065003 (2009)
Fig. 3

Verification between MHD and GK on AEs

- “A linear benchmark for a toroidal Alfvén eigenmode (TAE) is done with 11participating codes with a broad variation in the physical as well as the numerical models.”
  [Könies (NF 2018)]
- A reasonable agreement of around 20% has been found for the growth rates.
- Another benchmark work was also conducted by Taimourzadeh (NF 2019).



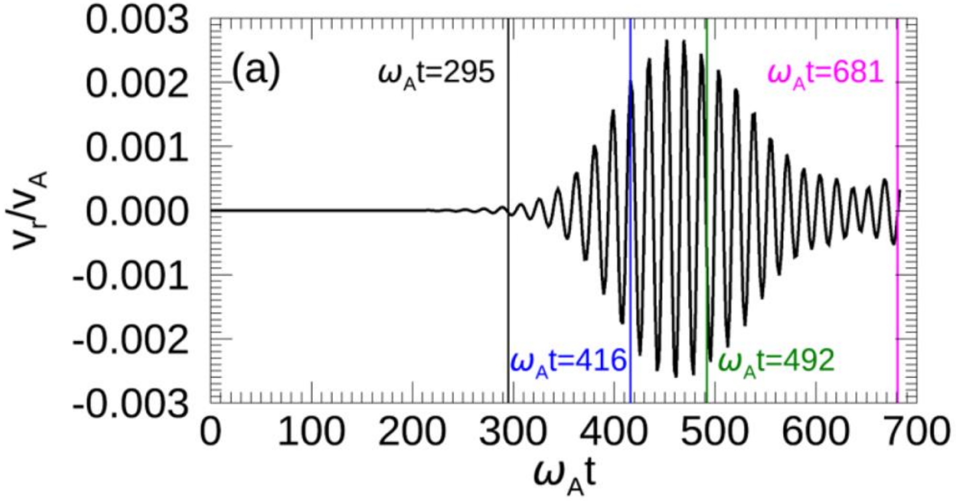
A. Könies et al.,
Nuclear Fusion **58**, 126027 (2018)

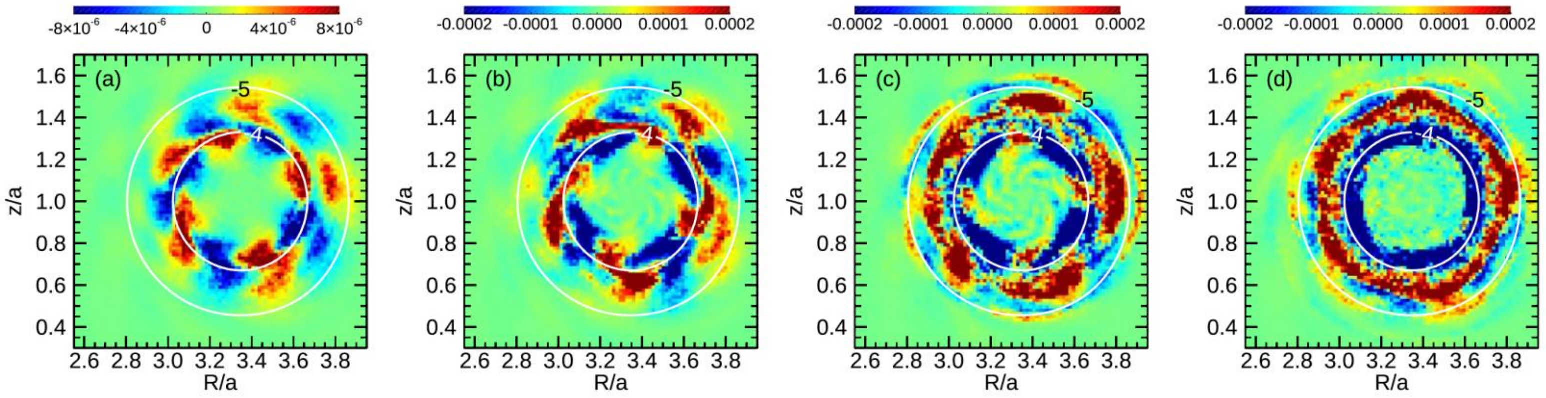
MEGA:
Hybrid simulation for energetic particles and MHD

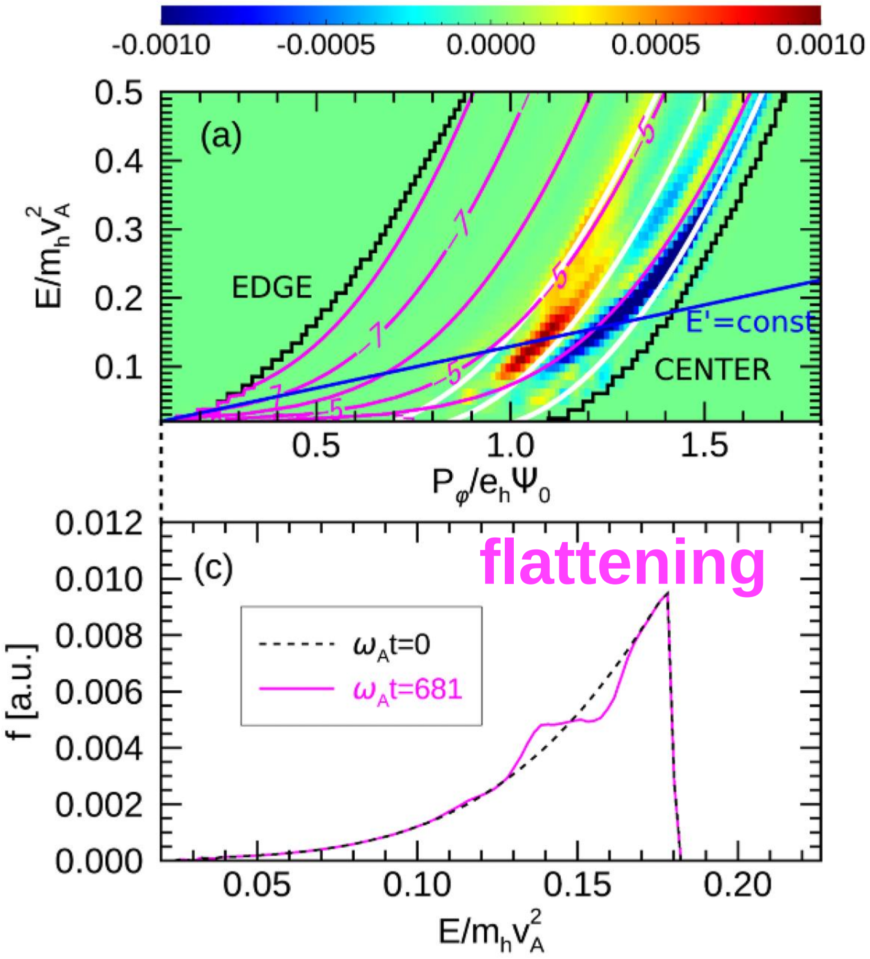
- energetic particles (fast ions, alphas, energetic electrons): gyrokinetic particle-in-cell (PIC) simulation
- bulk plasma: MHD simulation
- the coupling between EP and MHD is taken into account through the EP current in the MHD momentum equation
- **Extensions:**
  - **neutral beam injection (NBI), collisions, ICRF**
  - **multi-phase simulation** for fast ion distribution formation process in the slowing-down time scale
  - **kinetic thermal ions**

A single AE evolution and flattening of fast ion distribution function

(top L) evolution of AE
(bottom L) fast ion DF on a poloidal plane (R, z)
(top R) fast ion DF in (P_φ, E) plane
(bottom R) fast ion DF along an E'=const. line







Y. Todo et al., PPCF **63**, 075018 (2021)

NONLINEAR MHD EFFECTS AND ZF GENERATION

Y. Todo et al., Nucl. Fusion **50** (2010) 084016
Y. Todo et al., Nucl. Fusion **52** (2012) 094018

Comparison between linear and NL MHD runs (j_h' is restricted to n=4)

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho_{\mathrm{eq}}\mathbf{v}) + \nu_n \Delta(\rho - \rho_{\mathrm{eq}})$$

$$\rho_{\mathrm{eq}}\frac{\partial}{\partial t}\mathbf{v} = -\nabla p + (\mathbf{j}_{\mathrm{eq}} - \mathbf{j}'_{h\mathrm{eq}})\times\delta\mathbf{B} + (\delta\mathbf{j} - \delta\mathbf{j}'_h)\times\mathbf{B}_{\mathrm{eq}} + \frac{4}{3}\nabla(\nu\rho_{\mathrm{eq}}\nabla\cdot\mathbf{v}) - \nabla\times(\nu\rho_{\mathrm{eq}}\omega)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\frac{\partial p}{\partial t} = -\nabla\cdot(p_{\mathrm{eq}}\mathbf{v}) - (\gamma-1)p_{\mathrm{eq}}\nabla\cdot\mathbf{v} + \nu_n\Delta(p - p_{\mathrm{eq}}) + \eta\delta\mathbf{j}\cdot\mathbf{j}_{\mathrm{eq}}$$

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B}_{\mathrm{eq}} + \eta(\mathbf{j} - \mathbf{j}_{\mathrm{eq}})$$

$$\mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B}$$

$$\omega = \nabla\times\mathbf{v}$$

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}) + \nu_n \Delta(\rho - \rho_{\mathrm{eq}})$$

$$\rho\frac{\partial}{\partial t}\mathbf{v} = -\rho\omega\times\mathbf{v} - \rho\nabla\left(\frac{v^2}{2}\right) - \nabla p + (\mathbf{j} - \mathbf{j}'_h)\times\mathbf{B} + \frac{4}{3}\nabla(\nu\rho\nabla\cdot\mathbf{v}) - \nabla\times(\nu\rho\omega)$$

EP effects

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\frac{\partial p}{\partial t} = -\nabla\cdot(p\mathbf{v}) - (\gamma-1)p\nabla\cdot\mathbf{v} + \nu_n\Delta(p - p_{\mathrm{eq}}) + (\gamma-1)\left[\nu\rho\omega^2 + \frac{4}{3}\nu\rho(\nabla\cdot\mathbf{v})^2 + \eta\mathbf{j}\cdot(\mathbf{j} - \mathbf{j}_{\mathrm{eq}})\right]$$

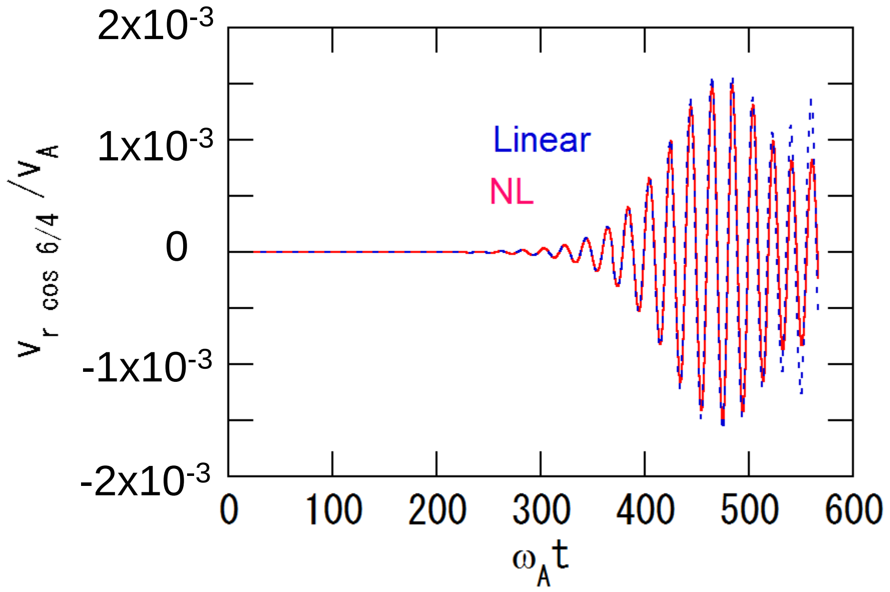
$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \eta(\mathbf{j} - \mathbf{j}_{\mathrm{eq}})$$

$$\mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B}$$

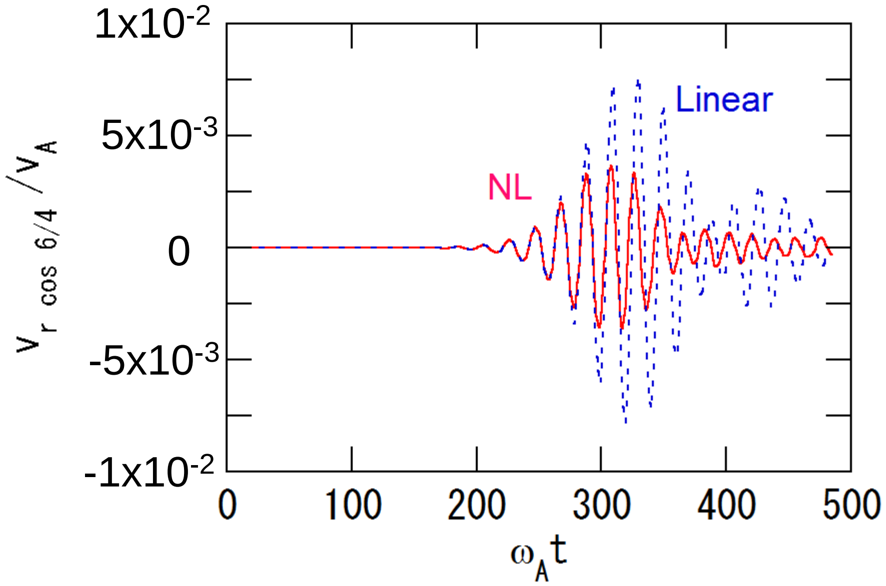
$$\omega = \nabla\times\mathbf{v}$$

The viscosity and resistivity are $\nu=\nu_n=2\times10^{-7}v_A R_0$ and $\eta=2\times10^{-7}\mu_0 v_A R_0$. The numbers of grid points are (128, 64, 128) for (R, φ, z). The number of marker particles is 5.2×10^5.

Nonlinear MHD effect on AE saturation level



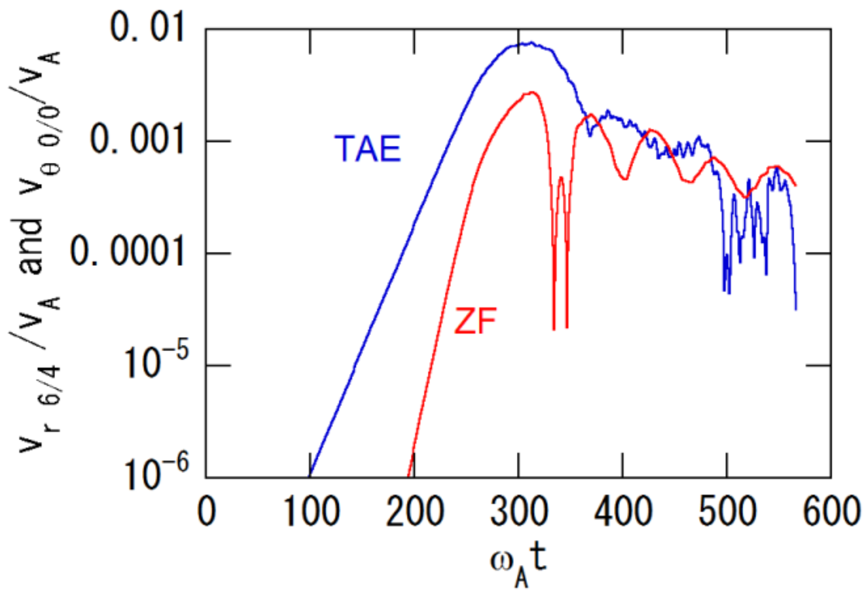
β_{h0}=1.5%
Sat. Level (linear) ~ 1.5x10^{-3}
Sat. Level (NL) ~ 1.5x10^{-3}

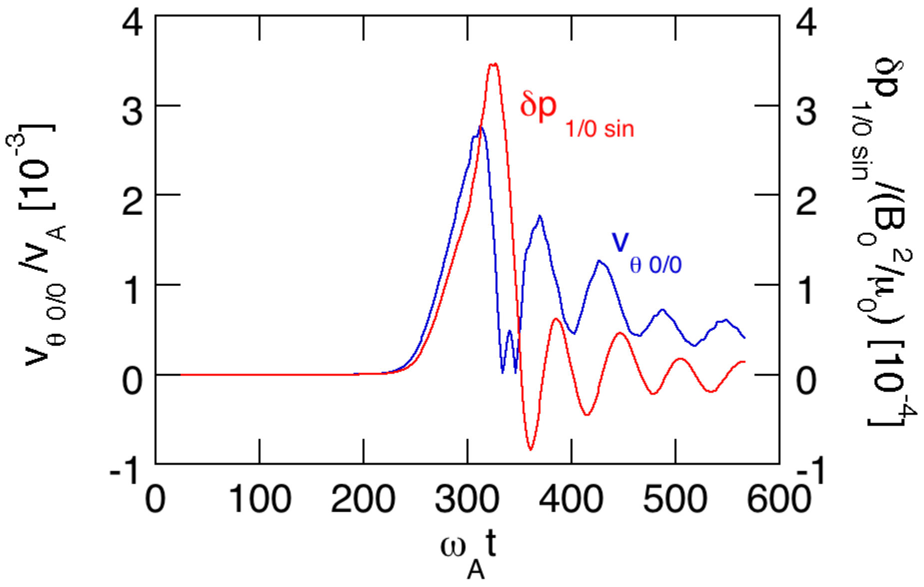


β_{h0}=2.0%
Sat. Level (linear) ~ 8x10^{-3}
Sat. Level (NL) ~ 4x10^{-3}

The saturation level is reduced to half by the nonlinear MHD effect.

ZF Evolution and GAM Excitation

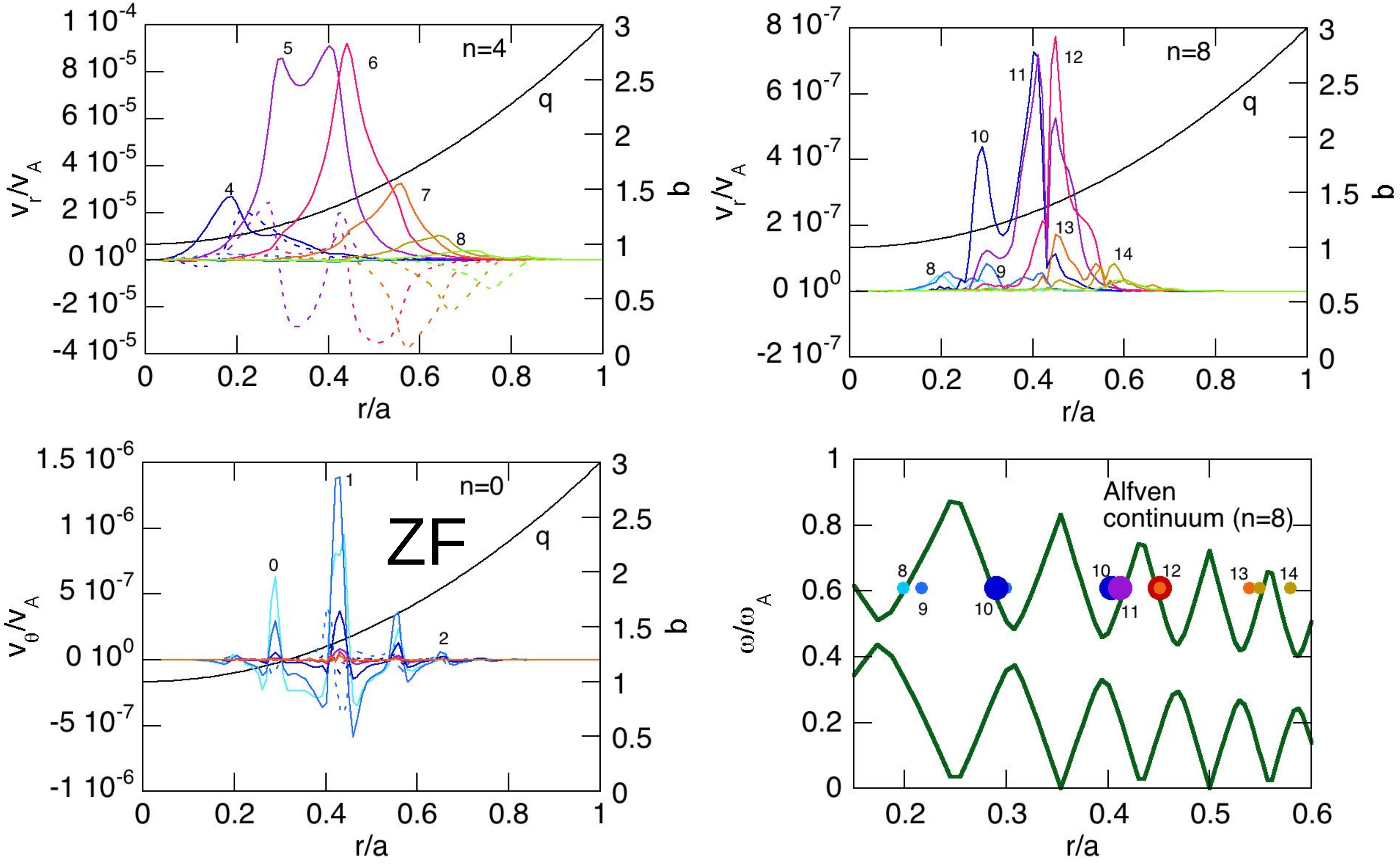




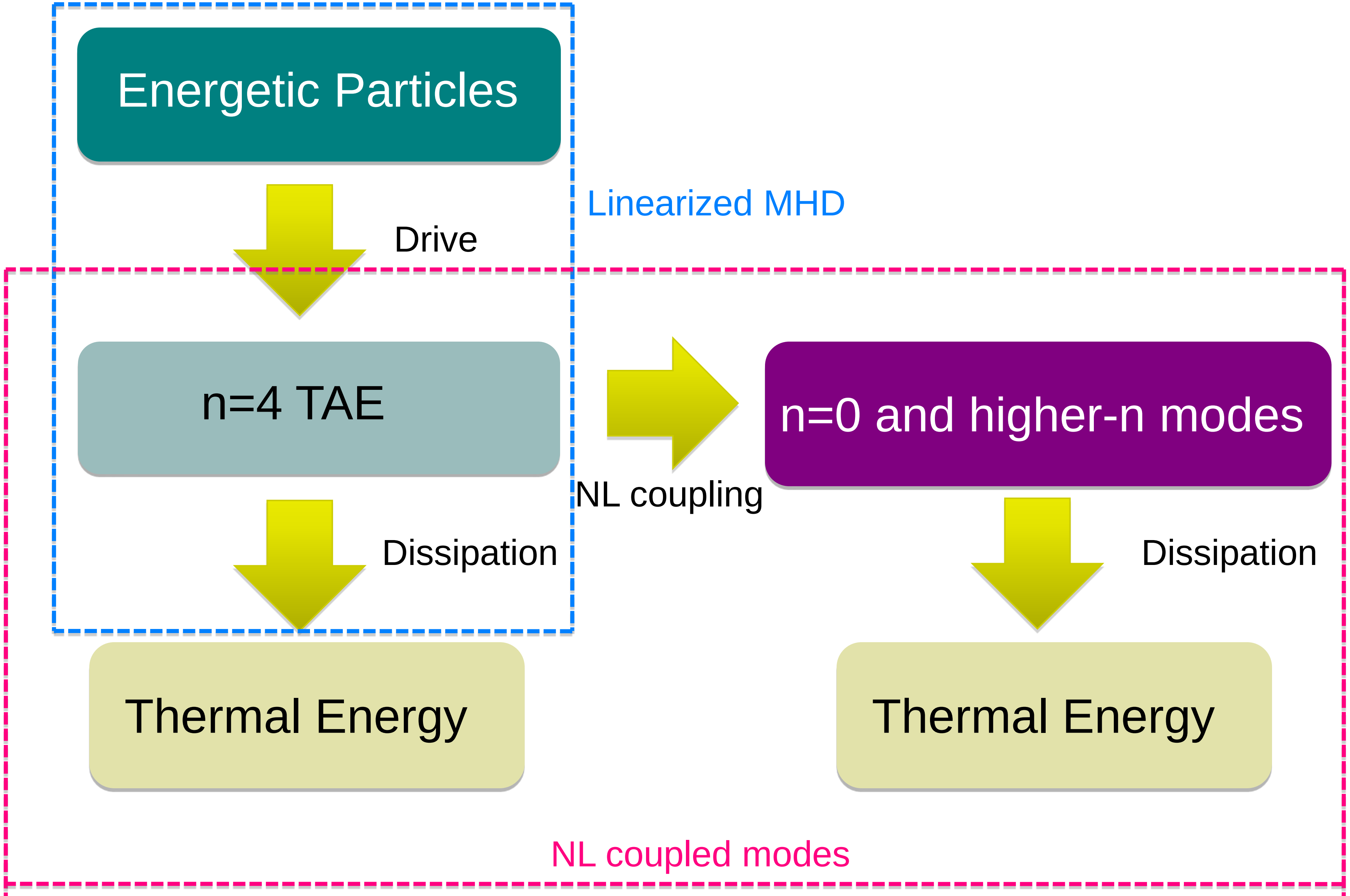
Evolution of TAE and zonal flow. The growth rate of ZF is twice that of the TAE.

After the saturation of the TAE instability, a geodesic acoustic mode (GAM) is excited.

Spatial profiles of the TAE and NL modes: Evidence for continuum damping of the higher-n (n=8) mode



Schematic Diagram of Energy Transfer



FAST ION PROFILE FLATTENING AND STIFFNESS IN DIII-D EXPERIMENTS

Y. Todo et al., Nucl. Fusion **54** (2014) 104012
Y. Todo et al., Nucl. Fusion **55** (2015) 073020
Y. Todo et al., Nucl. Fusion **56** (2016) 112008

Anomalous Flattening of Fast ion Profile on DIII-D

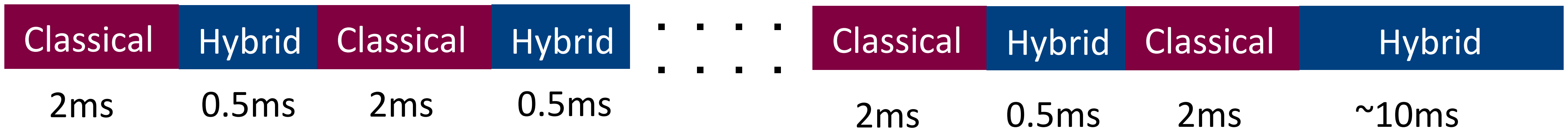
Fig. 1

Fig. 3

[W. W. Heidbrink, PRL **99**, 245002 (2007)]

- Anomalous flattening of the fast-ion profile during Alfvén-eigenmode activity
- A rich spectrum of TAEs and RSAEs with reversed q profile in current ramp-up phase

Multi-phase Simulation
[Y. Todo, Nucl. Fusion 54, 104012 (2014)]

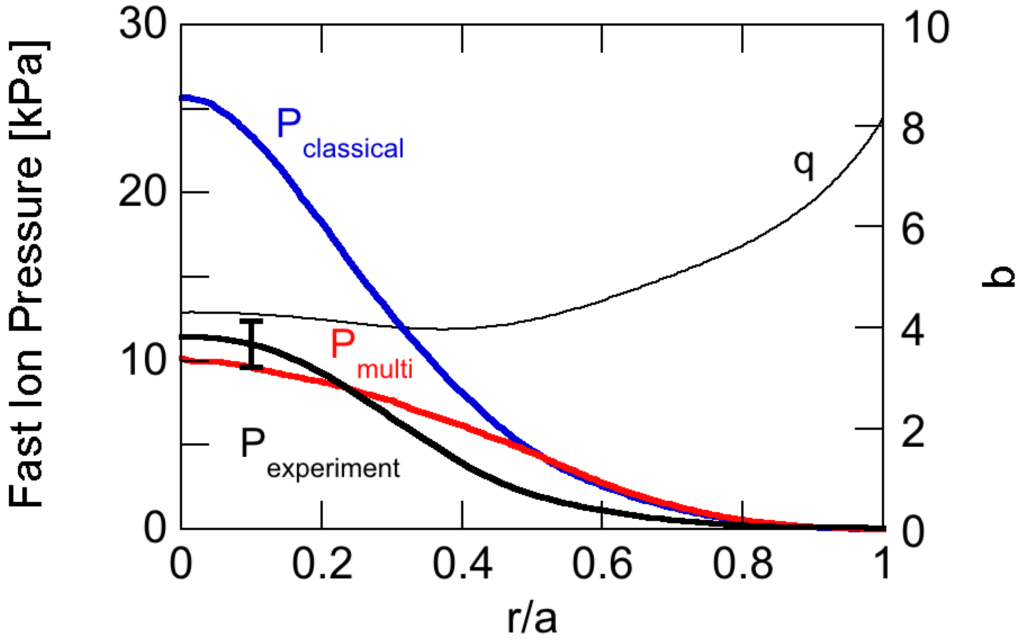


until a steady state or a limit cycle appears

- Hybrid simulation of energetic particles and an MHD fluid
- Multi-phase simulation =
  - classical simulation w/o MHD perturbations for 2ms +
  - EP-MHD hybrid simulation for 0.5ms; performed alternately
  - reduce computational time to 1/5

Comparison of fast ion pressure profiles (classical, multi-phase, exp.)

- Fast ion pressure profile flattening takes place in the multi phase simulation.
- The fast pressure profile in the multi-phase simulation is close to that in the experiment.



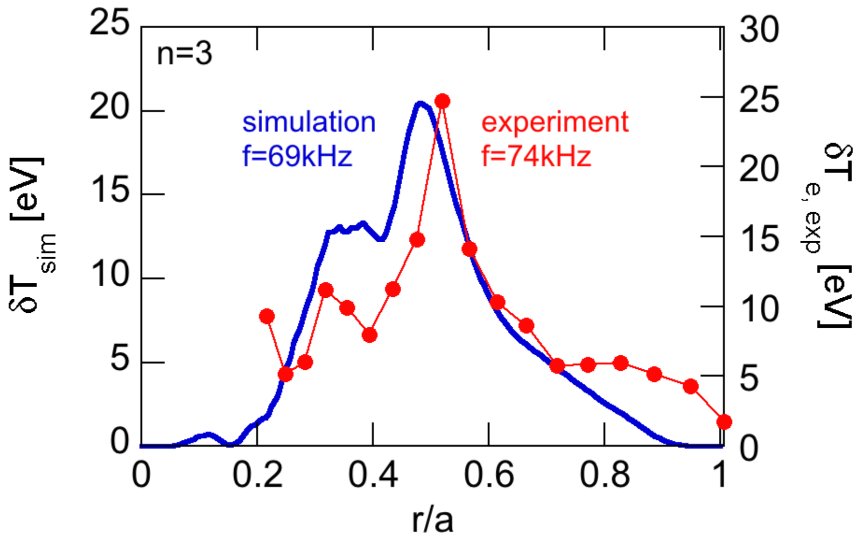
[Y. Todo et al., NF **55**, 073020 (2015)]

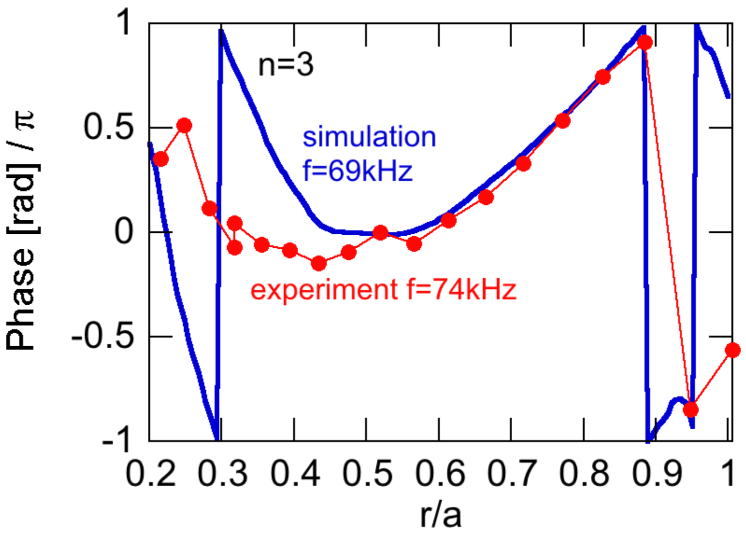
PLASMA SIMULATOR

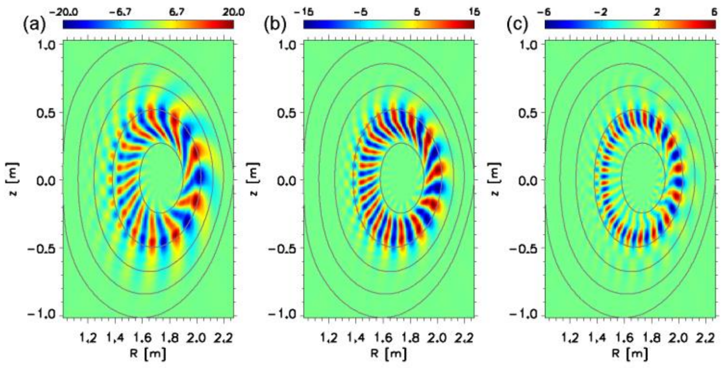
IFERC

DIII-D NATIONAL FUSION FACILITY

Comparison of temperature fluctuation profile with ECE measurement for n=3







[Y. Todo et al., NF **55**, 073020 (2015)]

- good agreement in spatial profile
- good agreement in **absolute amplitude**
- good agreement in phase profile

PLASMA SIMULATOR

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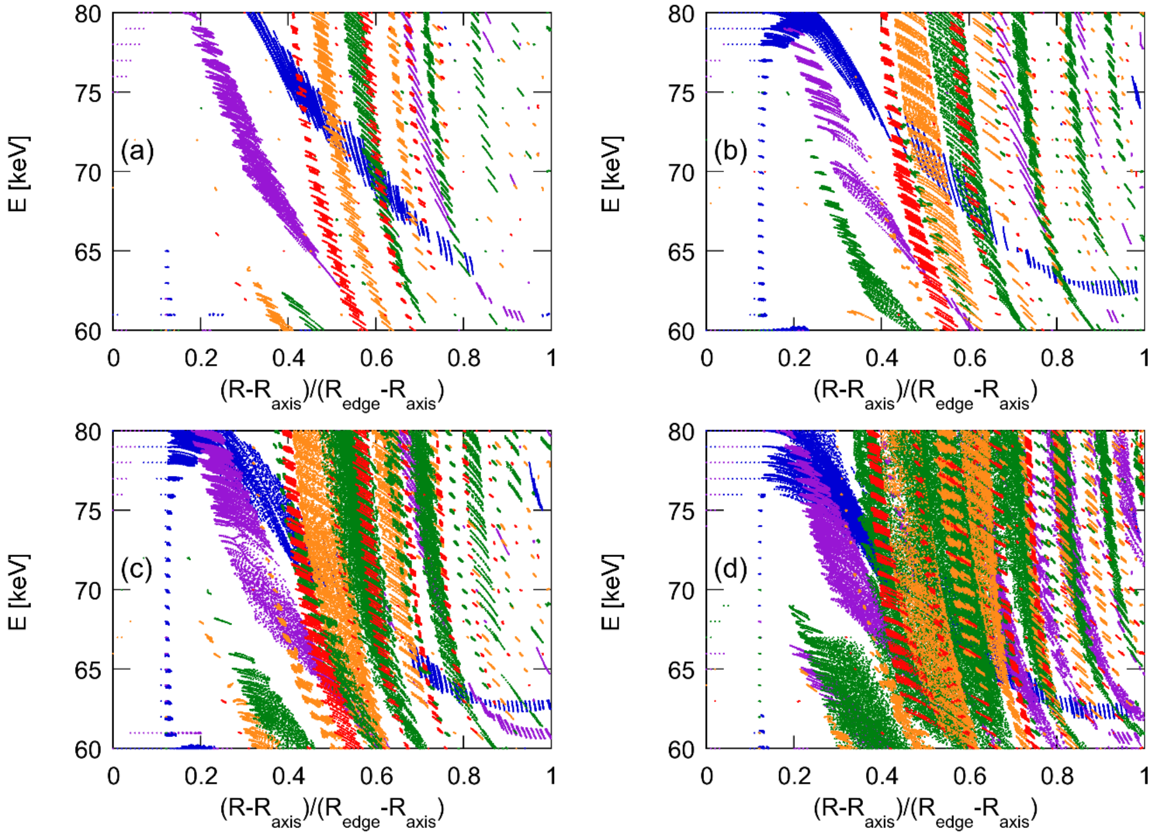
Resonance overlap in (R,E) phase space [blue (n=1), purple (n=2), green (n=3), orange (n=4), red(n=5)]

P_{NBI}=1.56MW

P_{NBI}=3.13MW



P_{NBI}=6.25MW

P_{NBI}=15.6MW

Phase space regions trapped by AEs (=resonances).
With increasing beam power [(a)->(d)], the resonance overlap covers the phase space.

[Y. Todo et al., Nucl. Fusion **56** (2016) 112008]

AE BURSTS & STEADY EVOLUTION, DEPENDENCE ON P_{NBI} AND SLOWING DOWN TIME

Y. Todo, New J. Physics **18**, 115005 (2016)
Y. Todo, Nucl. Fusion **59**, 096048 (2019)

Alfvén Eigenmode Bursts in TFTR

Fig. 4

Results from a TFTR experiment
[K. L. Wong et al., PRL **66**, 1874 (1991)]

Neutron emission:
nuclear reaction of thermal D and beam D
-> drop in neutron emission = fast ion loss

Mirnov coil signal:
magnetic field fluctuation
-> Alfvén eigenmode bursts

- Alfvén eigenmode bursts take place with a roughly constant time interval.
- 5-7% of energetic beam ions are lost at each burst.

AE Bursts have been observed in many tokamaks and stellarators/heliotrons

- tokamaks
  - DIII-D [Heidbrink+ NF1991]
  - JT-60U [Kusama+ NF1999]
  - NSTX [Fredrickson+ PoP2006]
  - MAST [Gryaznevich+ NF2006]
- stellarators/heliotrons
  - CHS/LHD [Toi+ NF2000]
  - W-7AS [Weller+ PoP2001]
  - TJ-II [Jiménez-Gómez+, NF2011]
  - Heliotron J [Yamamoto+ NF2017]

Fig. 9

Multiple AEs

AE bursts in LHD
[Osakabe+, NF **46**, S911 (2006)]

Resonance overlap of multiple modes; evolution of bump-on-tail instability with two waves

Stairway distribution

Fig. 7

Fig. 6

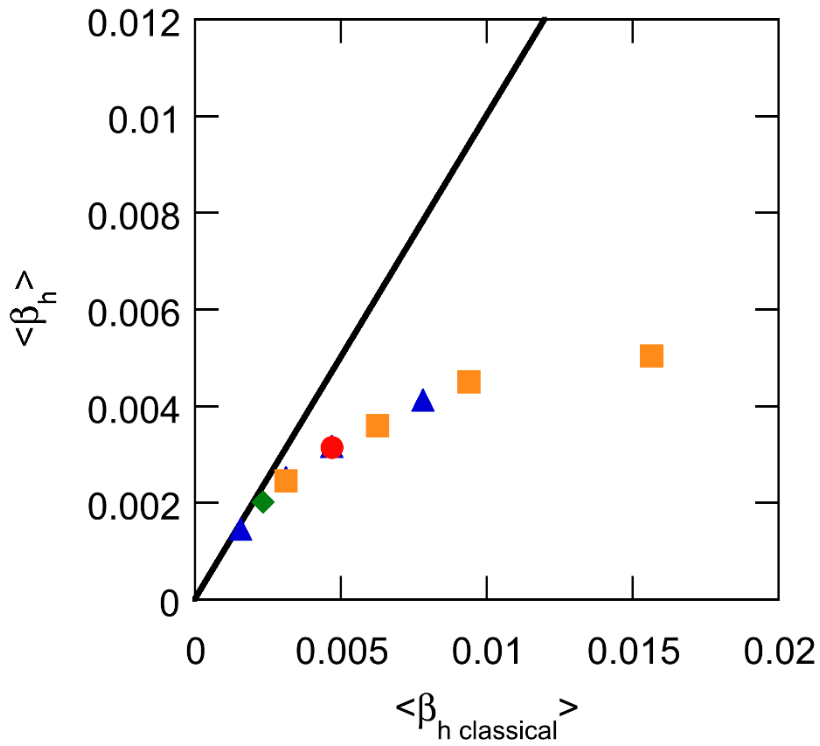
(top) EP distribution function for different moments with two waves.

(bottom) Time evolution of the total wave energy.

[Berk, Breizman et al., Phys. Plasmas **2**, 3007 (1995)]

Objective

- Apply the multi-phase hybrid simulation to a tokamak plasma similar to the TFTR experiment for various beam power and slowing down time
  - time evolution (steady, intermittent, time interval of bursts)
  - maximum amplitude
  - degradation of fast ion confinement
  - fast ion profile resiliency (saturation of fast ion pressure profile)
- Investigate the physical process of the AE bursts



Fast ion confinement degrades for higher classical fast ion beta.
[Y. Todo, New J. Phys. **18**, 115005 (2016)]

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Simulation condition based on the TFTR experiment

- R_0=2.4m, a=0.75m, B_0=1T
- beam injection energy 110keV (deuterium)
- n_i=2.8 x 10^{19} m^{-3} (deuterium)
- $q(r) = 1.2 + 1.8(r/a)^2$
- beam deposition profile:
  $\exp[-(r/0.4a)^2] \times \exp[-(|\lambda|-\lambda_0)^2/\Delta\lambda^2]$

$\lambda=v_{//}/v$, $\lambda_0=0.7$, $\Delta\lambda=0.3$

- slowing down & pitch-angle scattering

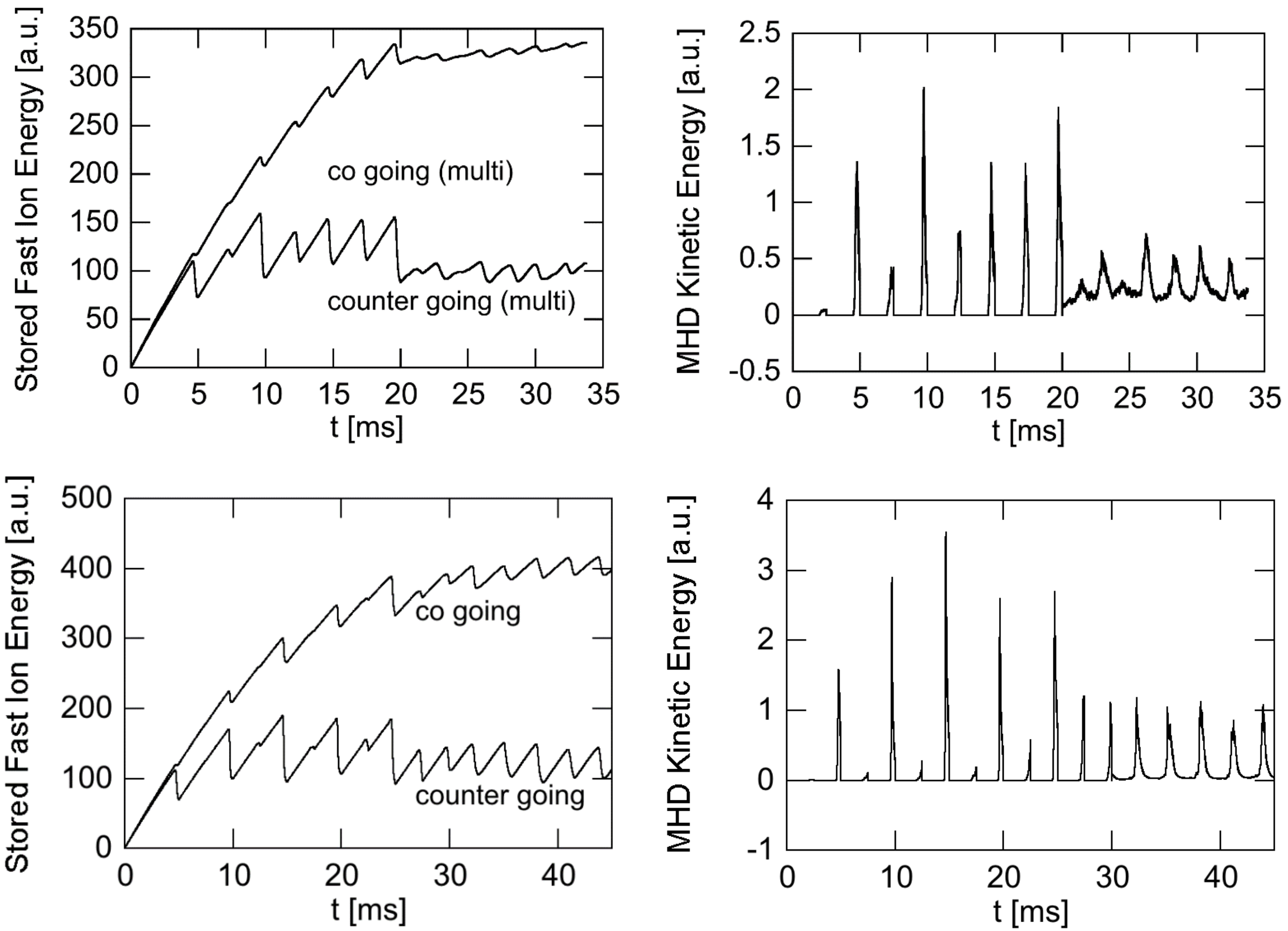
$v_d=(1/2)v_s\,(v_c/v)^3$

P_{NBI}=10MW, τ_s=100ms (similar to the TFTR experiment)

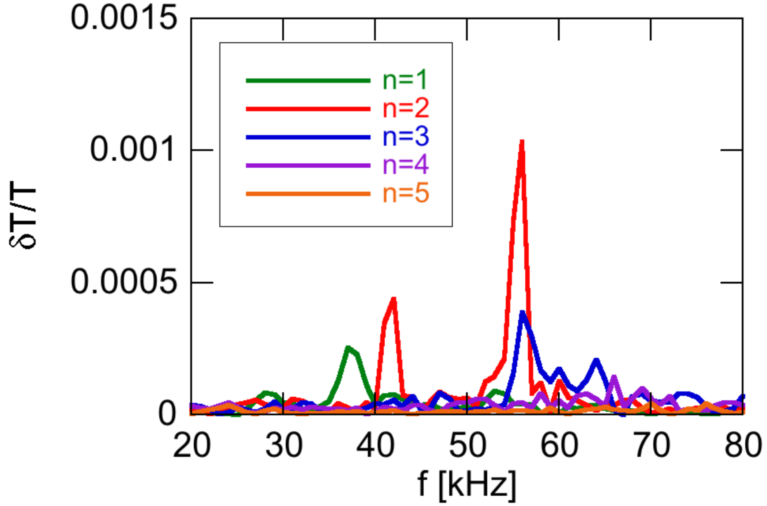
4.2M particles
[Y. Todo, New J. Phys. 2016]

for distribution function analysis -> 67M particles

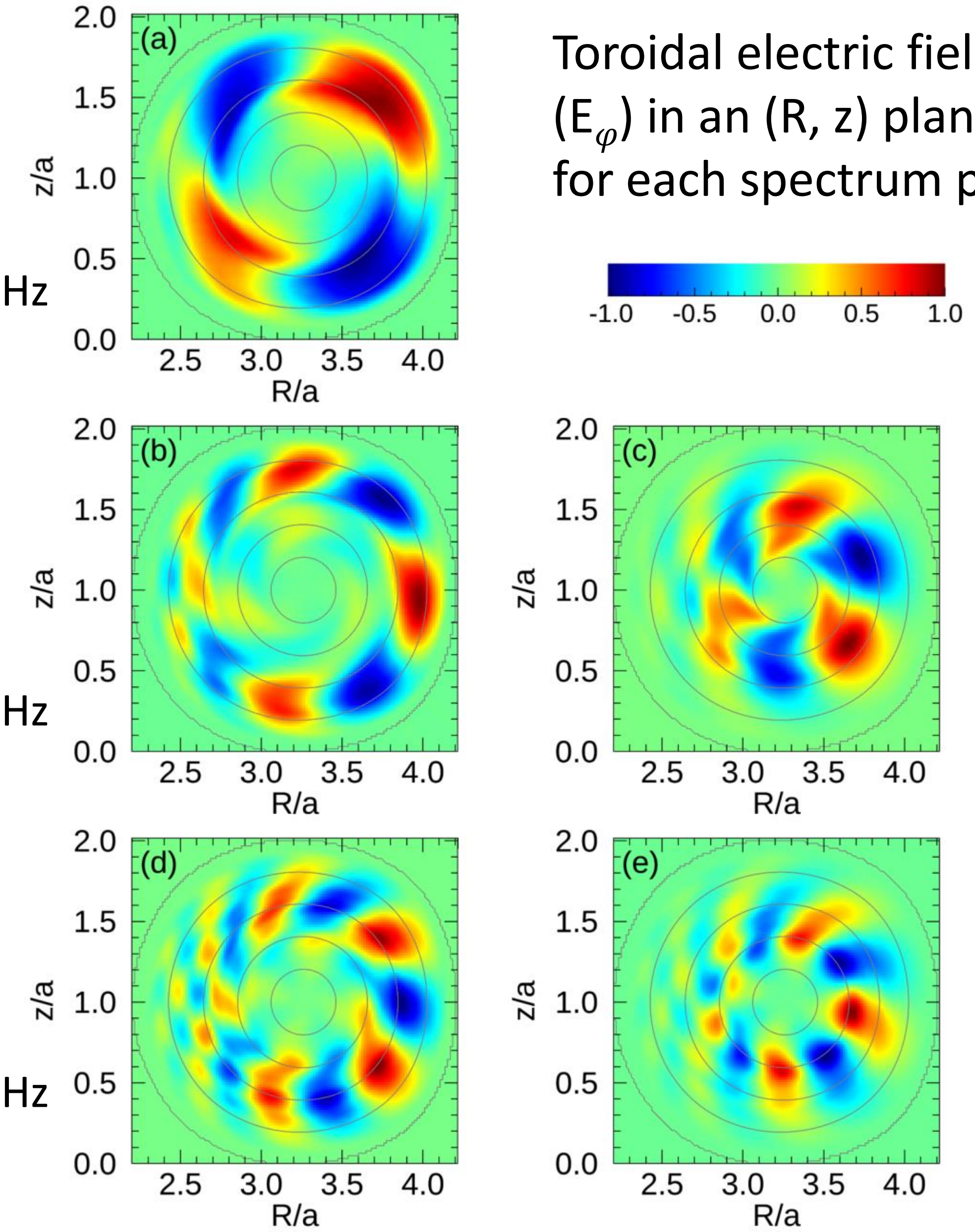
- Good convergence in number of particles
- Reduced numerical noise



Frequency spectra and spatial profiles of AEs



Toroidal electric field (E_φ) in an (R, z) plane for each spectrum peak



n=1, 37kHz

n=2, 42kHz

n=2, 56kHz

n=3, 56kHz

n=3, 64kHz

- AEs with n=1-5 are destabilized.
- The largest amplitude modes are n=1-3.
- The n=1 mode is an EPM, and the others are TAE.

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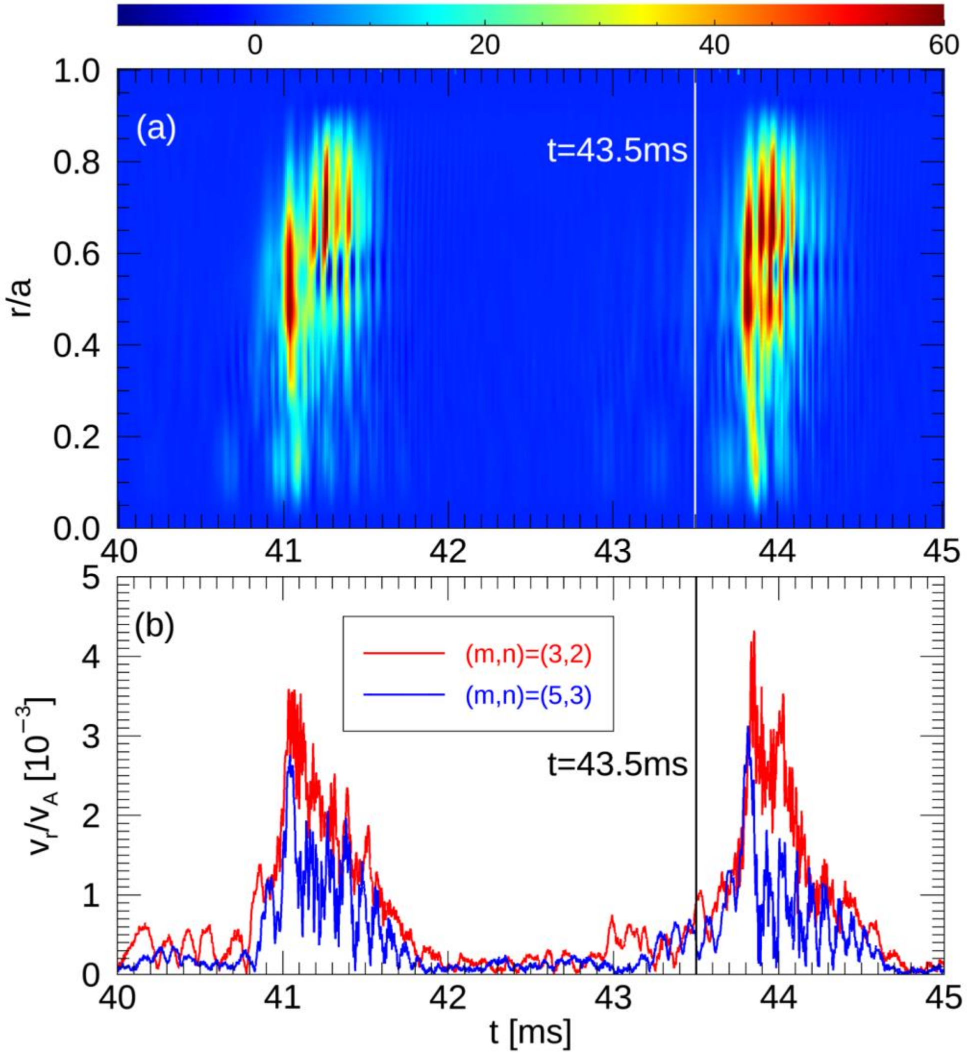
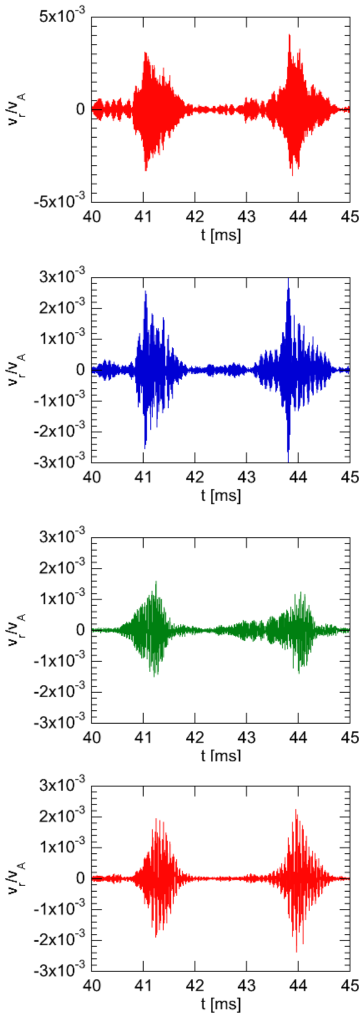
Synchronization of multiple AEs and fast ion energy flux profile evolution





m/n=3/2

m/n=5/3

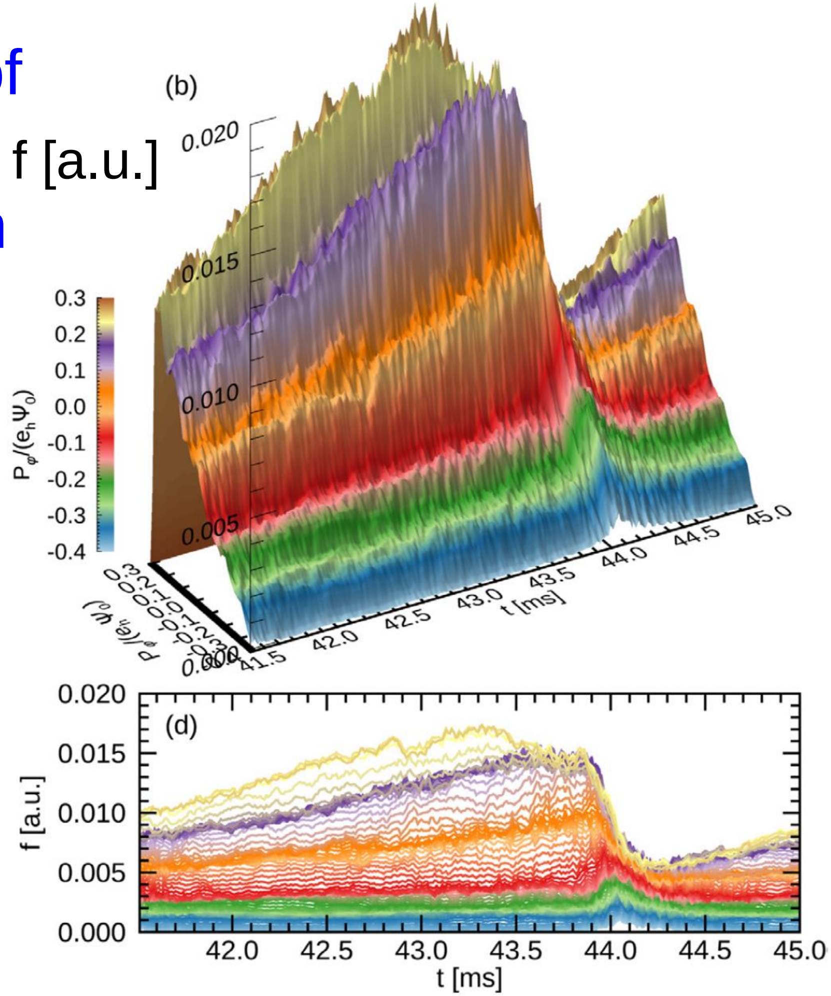
m/n=2/1

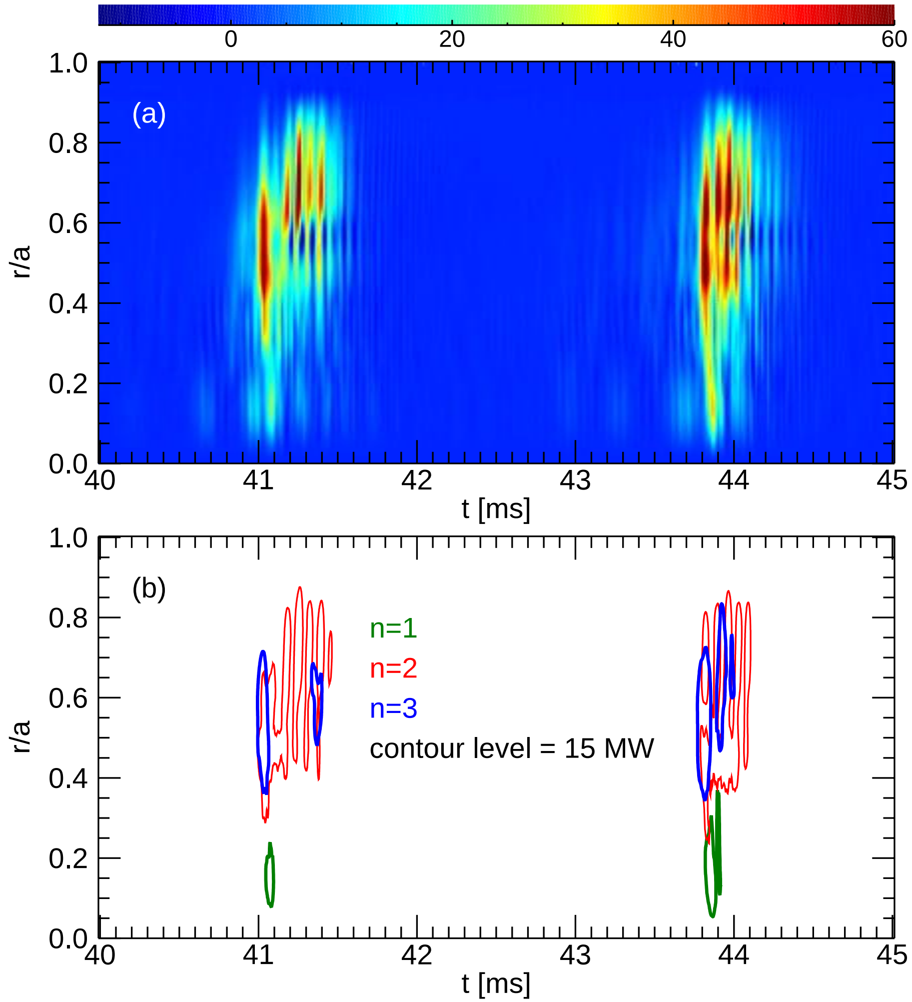
m/n=4/2

- Synchronization of multiple AEs.
- Time interval of the bursts is ~3ms.
- Maximum amplitude is $v/v_A \sim 3 \times 10^{-3}$.
  => close to the TFTR experiment.
- Energy flux ~ 60MW (NBI 10MW)
- Focus on the distribution function at t=43.5ms.

Collapse of stairway distribution and contribution of multiple modes to energy flux

collapse of stairway distribution





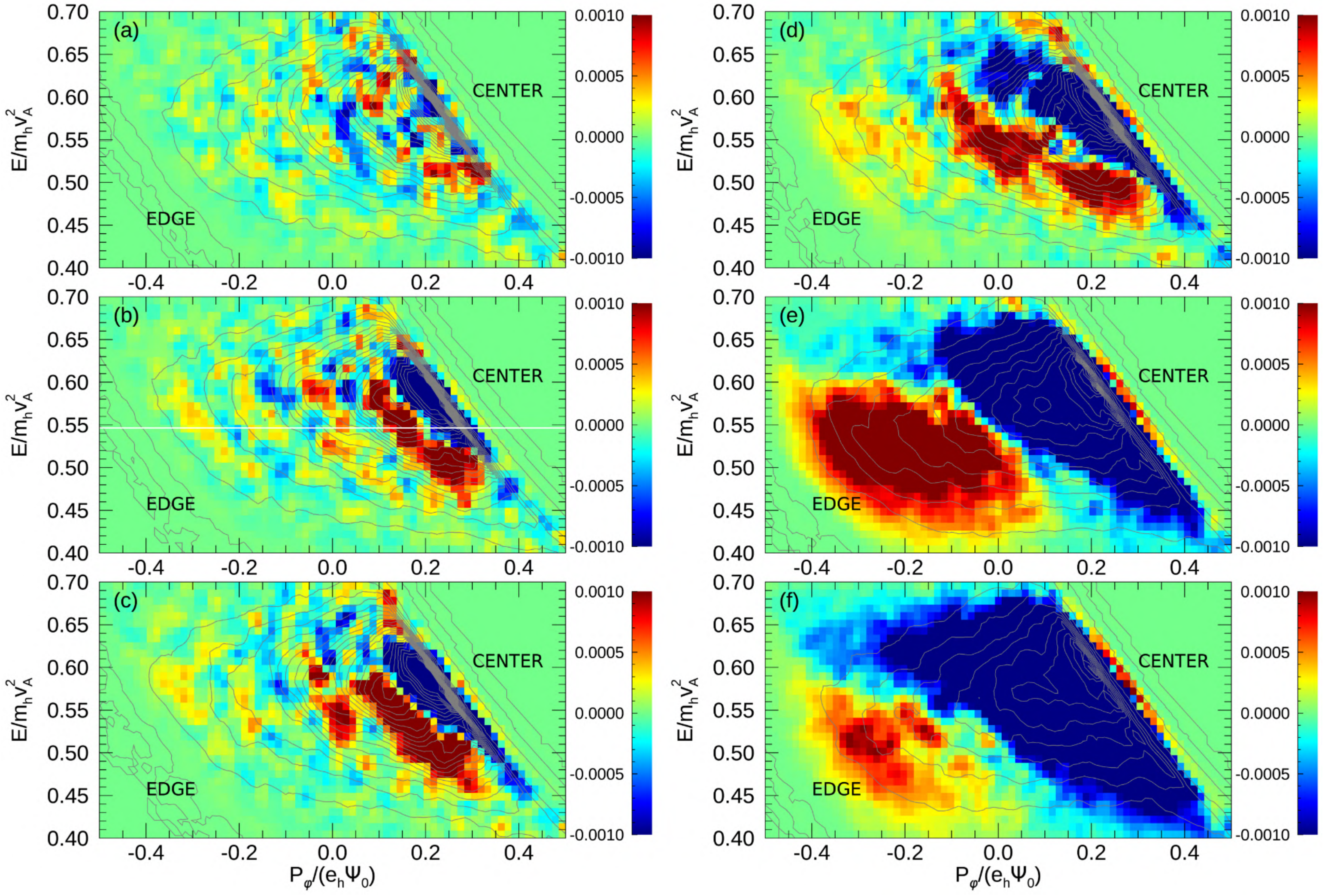
contribution of multiple modes to energy flux

Time evolution of $f(P_\varphi, E) - f_{classical}(P_\varphi, E)$ for constant μ (counter-going)

42.5ms

43.5ms

43.7ms



43.9ms

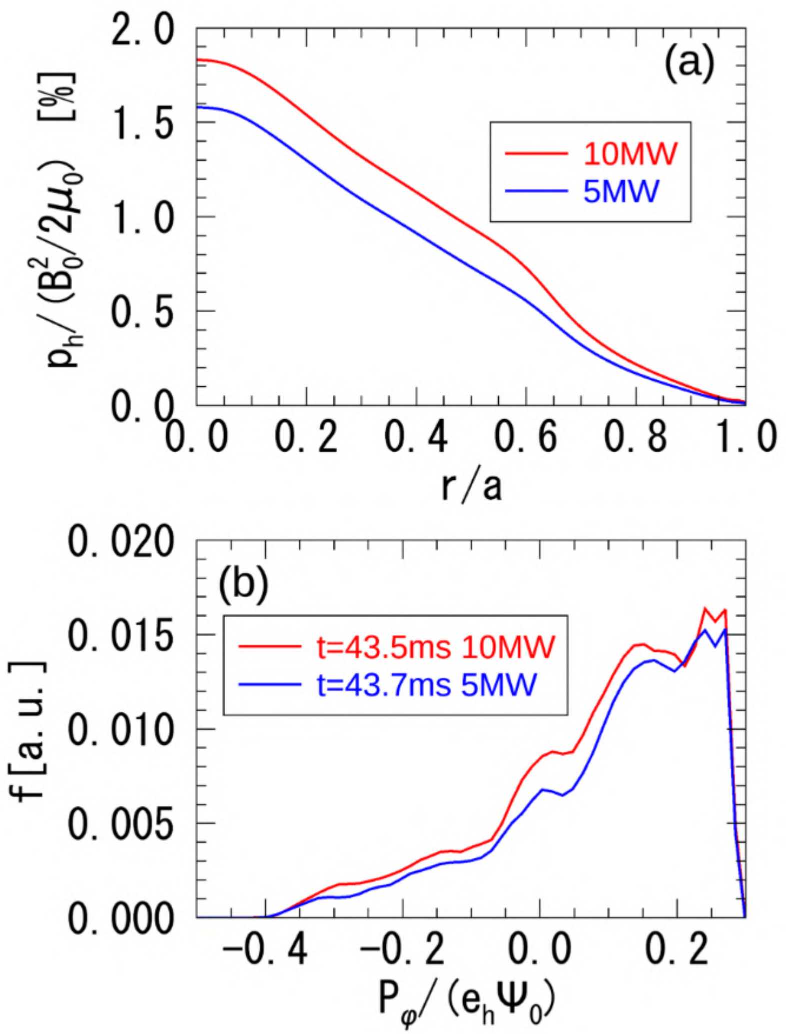
44.05ms

44.2ms

E

P_φ

Fast ion profile resiliency



(a) Fast ion pressure rises only by 15% for doubled beam power = **profile resiliency**.

(b) Profile resiliency is found also for fast ion **distribution function** just before the bursts.

Summary

Injection of energetic particles (NBI) with a constant power

Local flattening of distribution function due to Alfvén eigenmodes (AEs)
=
Stairway distribution

Constant NBI -> gradual increase in distribution

Critical distribution

Global resonance overlap
=
Trigger of AE burst

Collapse of the distribution

Summary of AE bursts (1)

- MEGA simulations of fast ion distribution formation process
  - for various beam deposition power (P_{NBI}) and slowing-down time (τ_s).
- With increasing volume-averaged classical fast ion pressure, the fast ion confinement degrades monotonically due to the transport by the Alfvén eigenmodes.
- For P_{NBI}=10MW and τ_s=100ms (similar to the TFTR experiment)
  - AE bursts (=synchronized sudden growth of multiple AEs) take place
  - with a time interval ~3ms and
  - the maximum amplitude $v_r/v_A=3 \times 10^{-3}$,
  - which are close to the TFTR experiment.

Summary of AE bursts (2)

- Before the sudden growth (=AE burst),
  - multiple AEs grow to low amplitude
  - low-amplitude AEs locally flatten the fast ion distribution
  - formation of a stairway distribution
- The stairway distribution ="critical distribution" where the further beam injection leads to
  - broadening of the locally flattened regions and their overlap
- The overlap of locally flattened regions (=resonance overlap) brings about
  - synchronized sudden growth of AEs and global transport of fast ions
  - profile resiliency = almost the same fast ion pressure profile and distribution function for 5MW and 10MW beam power

Summary

- Energetic particles (EPs）and Alfvén eigenmodes (AEs) in fusion plasmas
- Resonance condition, conserved quantity, and inverse Landau damping
- Phase space islands created by particle trapping and higher-order islands
- Nonlinear evolution of a bump-on-tail instability and frequency chirping
- Kinetic-MHD hybrid simulation
- Hybrid simulation for EP and MHD
  - Nonlinear MHD effects and zonal flow generation
  - Validation on DIII-D experiments (fast ion profile flattening and stiffness, electron temperature fluctuations)
  - AE burst and critical fast-ion distribution (profile resiliency)